1-14 Mutual Coupling between Antennas

The simplest approach for coupling between antennas is to start with a far-field approximation. We can modify Eq. (1-8) for path loss and add the phase term for the finite distance to determine the s-parameter coupling.

$$S_{21} = \sqrt{G_1 G_2}\,\frac{e^{-jkr}}{2kr}\,\frac{\mathbf{E_1}\cdot\mathbf{E}_2^*}{|\mathbf{E}_1||\mathbf{E}_2|} \tag{1-54}$$

Eq. (1-54) includes the polarization efficiency when the transmitted polarization does not match the receiving antenna polarization. We have an additional phase term because the signal travels from the radiation phase center along equivalent transmission lines to the terminals of each antenna. Equations (1-52) and (1-54) have the same accuracy except Eq. (1-54) eliminates the need to solve the two-port circuit matrix equation for transmission loss. These formulas assume antenna size is insignificant compared to the distance between the antennas, and each produces approximately uniform amplitude and phase fields over the second element.

When antennas are in the near field of each other, Eq. (1-54) can be modified so that radiation from different portions of a large antenna, such as a reflector, are divided into smaller portions that radiate in the far-field of the receiving antenna. We modify Eq. (1-54) to a summation.

$$S_{21} = \sum_i^N \sqrt{G_{1i} G_2}\,\frac{e^{-jkr_i}}{2kr_i}\,\frac{\mathbf{E_{1}}_i(\theta_i,\phi_i)\cdot\mathbf{E}_2^*(\theta_i,\phi_i)}{|\mathbf{E}_{1i}(\theta_i,\phi_i)||\mathbf{E}_2(\theta_i,\phi_i)|} \tag{1-54a}$$

Now the gain G_i and radiated electric field $\mathbf{E_{1i}}$ are for a portion of the antenna and the phase and range distance are different for each sub-element of the large antenna. The radiation from second antenna is in its far-field at the location of each small portion of the first antenna and we use the radiated far-field pattern. Instead of using normalized electric fields we can express Eq. (1-54a) in terms of radiated fields (V/m). Multiply the near-field electric field by distance r and the conversion factor $\sqrt{4\pi/(\eta P)}$ as in Eq. (1-4) to convert to the square root of gain.

$$S_{21} = \sum_i^N \frac{4\pi r_i^2 e^{-jkr_i}}{2kr_i\eta}\,\frac{\mathbf{E_{1}}_i(\theta_i,\phi_i)\cdot\mathbf{E}_2^*(\theta_i,\phi_i)}{\sqrt{P_1P_2}} = \sum_i^N \frac{\lambda r_i e^{-jkr_i}}{\eta}\,\frac{\mathbf{E_{1}}_i(\theta_i,\phi_i)\cdot\mathbf{E}_2^*(\theta_i,\phi_i)}{\sqrt{P_1P_2}}$$

Now the electric fields are near-fields (V/m) radiated at the center of each antenna from the other. If both antennas are large, we apply a double summation and use the range difference $r_i - r_j$.

$$S_{21} = \sum_i^N \sum_j^M \frac{\lambda r_{i-j} e^{-jkr_{i-j}}}{\eta}\,\frac{\mathbf{E_{1}}_i(\theta_i,\phi_i)\cdot\mathbf{E}_{2j}^*(\theta_j,\phi_j)}{\sqrt{P_1P_2}}$$

We use Eq. (1-54a) for ray tracing methods when multiple rays reach the receiving antenna from multiple directions and near field formulations are used for all rays.

An example of using this method is the coupling between two reflector antennas. One reflector feed transmits and the other receives. The transmitting feed radiates directly to the receiving feed. The transmitting feed radiates and excites currents on both reflectors (a physical optics approach) and can include rim PTD currents. The currents excited on both reflectors are subdivided into small patches whose radiation is in the far-field of the receiving feed. The radiation from each patch is included in the sum (1-54a). Additional terms of the currents excited on each reflector by the currents initially excited by the feed on the other reflector can be added to small patches in the far field of the receiving feed can also be added. If the receiving reflector is not in the main beam of the transmitting reflector, the GTD (geometric theory of diffraction) method can be used. GTD requires that we account for blockage of direct radiation while physical optics analysis does not need to account for blockage. We start with direct radiation between the two feeds. We add the singly diffracted rim terms from both reflectors for rays directly into the receiving feed and include effect of its radiation pattern in the direction of the rim diffracted ray. When we add the doubly diffracted rays from the two reflectors, we include the effect of the interaction of the two reflectors provided we weight the pattern of the receiving antenna in the direction of the doubly diffracted ray. Eq. (1-54a) is suitable for both physical optics and ray tracing methods while the method below requires using currents excited on scatterers (physical optics or method of moments). This describes a method where radiation terms are added one at a time because it is best to determine the each effect and not include what could be insignificant multipath terms (Section 1-17).

We can improve on Eq. (1-54) when we use the current distribution on one of the two antennas and calculate the near-field fields radiated by the second antenna at the location of these currents. Since currents vary across the receiving antenna, we use vector current densities to include direction: $\mathbf{J}_\mathbf{r}$ electric and $\mathbf{M}_\mathbf{r}$ magnetic. Although magnetic current densities are fictitious, they simplify the representation of some antennas. We compute coupling from reactance, an integral across these currents [see Eq. (2-34)].

$$S_{21} = \frac{j}{2\sqrt{P_r P_t}} \iiint (\mathbf{E}_{\boldsymbol{\tau}} \cdot \mathbf{J}_\mathbf{r} - \mathbf{H}_\mathbf{t} \cdot \mathbf{M}_\mathbf{r}) dV \qquad (1\text{-}55)$$

The input power to the transmitting antenna P_t produces fields $\mathbf{E}_\mathbf{t}$ and $\mathbf{H}_\mathbf{t}$**.** The power P_r into the receiving antenna excites the currents. The scalar product between the incident fields and the currents includes polarization efficiency. If we know the currents on the transmitting antennas, we calculate the near-field pattern response from them at the location of the receiving antenna. Similar to many integrals, Eq. (1-55) is notional because we perform the integral operations only where currents exist. The currents could be on wire segments or surfaces. A practical implementation of Eq. (1-55) divides the currents into patches or line segments and performs the scalar products between the currents and fields on each patch and sums the result. A second form of the reactance [see Eq. (2-35)] involves an integral over a surface surrounding the receiving antenna. In this case each antenna radiates its field to this surface, which requires near-field pattern calculations for both. Equation (1-55) requires adding the phase length between the input ports and the currents, similar to using Eq. (1-54). When we use Eq. (1-55), we assume that radiation between the two antennas excites insignificant additional currents on each other. We improve the answer by using a few iterations of physical optics, which finds induced currents from incident fields (Chapter 2).

We improve on Eq. (1-55) by performing a moment method calculation between the two antennas. This involves subdividing each antenna into small elements excited with simple assumed current densities. Notice the similarity between Eqs. (1-52) and (1-54) and realize Eq. (1-55) is a near-field version of Eq. (1-54). We use reactance to compute the mutual impedance Z_{21} between the small elements as well as their self impedance. For the moment method we calculate a mutual impedance matrix with a row and column for each small current element. We formulate a matrix equation using the mutual impedance matrix and an excitation vector to reduce coupling to a circuit problem. This method includes the additional currents excited on each antenna due to the radiation of the other.