2-4.5 Huygens sources on apertures

In section 2-2.2 we considered a plane wave incident normal to the *x-y* plane and used the induction theorem to replace this incident wave with integrals over the aperture electric and magnetic fields to obtain the far-field pattern Eqs. (2-22) and (2-24). This was a special case because the incident wave propagation was normal to the aperture. We will show that the Huygens source method is equivalent to physical optics and we have our choice of integrating over the fields or current using the Green's function.

Suppose the incident wave approaches the *x-y* plane at an angle α in the *x-z* plane. E_y is the perpendicular polarization and H_x is the parallel polarization component. For perpendicular polarization the magnetic field has both H_x and H_z components, while the electric field has both E_x and E_z components for parallel polarization. We require the fields parallel to the surface to calculate the equivalent electric and magnetic currents in the aperture.

For perpendicular polarization, we obtain the magnetic field parallel to the surface is computed from E_y assuming the ratio of the electric and magnetic field equals the free space impedance η.

$$H_x = -\frac{\cos\alpha}{\eta} E_y \text{ perpendicular polarization}$$

While we compute the electric field in the plane of the parallel polarization using the equation

$$E_x = \eta\cos\alpha H_y \text{ or } H_y = \frac{E_x}{\eta\cos\alpha} \text{ parallel polarization}$$

We use Eqs. (2-22) to integrate across the aperture to compute the far-field radiation, which we can combine because we assumed the ratio of the electric and magnetic fields is η in the incident waves.

$$-\eta g_x = f_y \cos\alpha \text{ and } \eta g_y = \frac{f_x}{\cos\alpha}$$

We collect terms similar to Eq. (2-24) to obtain expressions for the far-field radiation.

$$E_\theta = \frac{jke^{-jkr}}{4\pi r}\left[f_x \cos\phi\left(1+\cos\theta\cos\alpha\right) + f_y \sin\phi\left(1+\frac{\cos\theta}{\cos\alpha}\right)\right]$$

$$E_\phi = \frac{-jke^{-jkr}}{4\pi r\cos\alpha}\left[f_x \sin\phi\left(1+\cos\theta\cos\alpha\right) - f_y \cos\phi\left(1+\frac{\cos\theta}{\cos\alpha}\right)\right]$$

In terms of a physical optics currents the currents in the aperture can be expressed in terms of the parallel and perpendicular components.

$$\mathbf{E} \text{ x } \hat{\mathbf{n}} = \mathbf{M} = E_\perp \hat{\mathbf{x}} - E_\parallel \hat{\mathbf{y}}$$

$$\hat{\mathbf{n}} \text{ x } \mathbf{H} = \mathbf{J} = \frac{E_\parallel}{\eta\cos\alpha}\hat{\mathbf{x}} + \frac{E_\perp \cos\alpha}{\eta}\hat{\mathbf{y}}$$

Aperture on Cylinder

Given the aperture fields on a differential point on a cylinder E_z', E_φ', H_z', H_φ', we calculate the radiated field by first finding equivalent electric and magnetic currents on the differential point and second either using the Green's function of physical optics or electric and magnetic vector potentials.

$$\mathbf{M_s} = \mathbf{E_a} \mathbf{x}\ \hat{\mathbf{n}} = (E_\phi'\hat{\mathbf{a}}_\phi' + E_z'\hat{\mathbf{a}}_\mathbf{z}) \times \mathbf{a}_\mathbf{r}' = -E_\phi'\hat{\mathbf{a}}_\mathbf{z} + E_z'\hat{\mathbf{a}}_\phi'$$

$$\mathbf{J_s} = \hat{\mathbf{n}} \times \mathbf{H_a} = \mathbf{a}_\mathbf{r}' \times (H_\phi'\hat{\mathbf{a}}_\phi' + H_z'\hat{\mathbf{a}}_\mathbf{z}) = H_\phi'\hat{\mathbf{a}}_\mathbf{z} - H_z'\hat{\mathbf{a}}_\phi'$$

The differential electric vector potential is

$$d\mathbf{F} = \frac{e^{-jkr}}{4\pi r} \mathrm{M}_{\mathbf{s}} a d\phi' dz' e^{j\mathbf{k}\cdot\rho} \text{ where } \rho = (\cos\phi' \hat{\mathbf{a}}_{\mathbf{x}} + \sin\phi' \hat{\mathbf{a}}_y) a + z\hat{\mathbf{a}}_z$$

$$\mathbf{k} = \frac{2\pi}{\lambda}\left(\sin\theta\cos\phi\hat{\mathbf{a}}_{\mathbf{x}} + \sin\theta\sin\phi\hat{\mathbf{a}}_y + \cos\theta\hat{\mathbf{a}}_z\right)$$

$$\mathbf{k}\cdot\rho = \frac{2\pi}{\lambda}\left[a\sin\theta\cos\left(\phi-\phi'\right) + z\cos\theta\right]$$

$$d\mathbf{F} = \frac{e^{-jkr}}{4\pi r}\left(E_z'\hat{\mathbf{a}}_\phi' - E_\phi'\hat{\mathbf{a}}_{\mathbf{z}}\right) a d\phi' dz' e^{j\frac{2\pi}{\lambda}\left[a\sin\theta\cos\left(\phi-\phi'\right)+z\cos\theta\right]}$$

$$d\mathbf{A} = \frac{-e^{-jkr}}{4\pi r}\left(H_z'\hat{\mathbf{a}}_\phi' - H_\phi'\hat{\mathbf{a}}_{\mathbf{z}}\right) a d\phi' dz' e^{j\frac{2\pi}{\lambda}\left[a\sin\theta\cos\left(\phi-\phi'\right)+z\cos\theta\right]}$$

The source unit vectors are projected (dot product) on the field coordinates. The radiated field E is the combination of the radiations from each vector potential.

$$d\mathbf{E} = -j\omega\mu d\mathbf{A} - j\eta\omega\varepsilon d\mathbf{F}\times\hat{\mathbf{a}}_{\mathbf{r}}$$

$$dE_\theta = -jk(\eta dA_\theta + dF_\phi) \text{ and } dE_\phi = -jk(\eta dA_\phi - dF_\theta)$$

By assuming a Huygens source at each point of the aperture, we can obtain the magnetic field from the electric field.

$$-\eta H_\phi = E_z \text{ and } \eta H_z = E_\phi$$

When we collect terms, the differential electric fields computed from the aperture fields become as follows.

$$dE_\theta = \frac{jke^{-jkr}}{4\pi r} a\left[E_\phi'\cos\theta\sin\left(\phi-\phi'\right) - E_z'\left(\cos\left(\phi-\phi'\right)+\sin\theta\right)\right] e^{jk\left[a\sin\theta\cos\left(\phi-\phi'\right)+z\cos\theta\right]} dz' d\phi'$$

$$dE_\phi = \frac{-jke^{-jkr}}{4\pi r} a\left[E_\phi'\left(\cos\left(\phi-\phi'\right)+\sin\theta\right) + E_z'\cos\theta\sin\left(\phi-\phi'\right)\right] e^{jk\left[a\sin\theta\cos\left(\phi-\phi'\right)+z\cos\theta\right]} dz' d\phi'$$

Aperture on Sphere

For a given source point on a sphere the magnetic and electric currents are computed from the expressions.

$$\mathbf{M}_{\mathbf{s}} = \mathbf{E}_{\mathbf{a}}\times\hat{\mathbf{n}} = \left(E_\theta'\hat{\mathbf{a}}_\theta' + E_\phi'\hat{\mathbf{a}}_\phi'\right)\times\hat{\mathbf{a}}_{\mathbf{r}}' = -E_\theta'\hat{\mathbf{a}}_\phi' + E_\phi'\hat{\mathbf{a}}_\theta'$$

$$\mathbf{J}_{\mathbf{s}} = \hat{\mathbf{n}}\times\mathbf{H}_{\mathbf{a}} = \hat{\mathbf{a}}_{\mathbf{r}}'\times\left(H_\theta'\hat{\mathbf{a}}_\theta' + H_\phi'\hat{\mathbf{a}}_\phi'\right) = H_\theta'\hat{\mathbf{a}}_\phi' - H_\phi'\hat{\mathbf{a}}_\theta'$$

We obtain the differential electric potential from the aperture point.

$$d\mathbf{F} = \frac{e^{-jkr}}{4\pi r}\mathbf{M}_{\mathbf{s}}\rho^2\sin\theta' e^{-j\mathbf{k}\cdot\rho} d\theta' d\phi'$$

$$\mathbf{k}\cdot\rho = \frac{2\pi}{\lambda}\left[\sin\theta\sin\theta'\cos\left(\phi-\phi'\right) + \cos\theta\cos\theta'\right]$$

The differential magnetic vector has the same form except of the exchange of electric current for magnetic current. The radiated electric field is computed from the same weighted sum of the potentials.

$$d\mathbf{E} = -j\omega\mu d\mathbf{A} - j\eta\omega\varepsilon d\mathbf{F}\times\hat{\mathbf{a}}_{\mathbf{r}}$$

$$dE_\theta = -jk(\eta dA_\theta + dF_\phi) \text{ and } dE_\phi = -jk(\eta dA_\phi - dF_\theta)$$

We apply the Huygens source approximation.

$$\eta H_\phi = E_\theta \text{ and } -\eta H_\theta = E_\phi$$

We collect terms and obtain the common term u for both expressions of differential fields.

$$u = \frac{jke^{-jkr}}{4\pi r}\rho^2 \sin\theta' e^{jk\rho\left(\sin\theta\sin\theta'\cos\left(\phi-\phi'\right)+\cos\theta\cos\theta'\right)}$$

The differential radiated fields from the spherical aperture source point are as follows.

$$dE_\theta = u\left\{E'_\theta\left[\cos\left(\phi-\phi'\right)\left(1+\cos\theta\cos\theta'\right)+\sin\theta\sin\theta'\right]+E'_\phi\sin\left(\phi-\phi'\right)\left(\cos\theta+\cos\theta'\right)\right\}$$

$$dE_\phi = -u\left\{E'_\theta\sin\left(\phi-\phi'\right)\left(\cos\theta-\cos\theta'\right)-E'_\phi\left[\cos\left(\phi-\phi'\right)\left(\cos\theta\cos\theta'-1\right)+\sin\theta\sin\theta'\right]\right\}$$

Integration over an aperture is a ray technique compatible to with GTD, although the patches used in PO can be used with GTD in the same manner. Physical optics allows the calculation of near field points because full near-field Green’s functions are available and easily implemented.