4-26 Aperture Approximation of Directivity given Beamwidths

By estimating the total integral of the radiated pattern, Kraus [see Section 1-8] devised a method of estimating directivity for pencil beam patterns with its peak at $\theta = 0°$. Given the half-power beamwidths of the principal plane patterns, the integral is approximately the product of the beamwidths. This idea comes from circuit theory, where the integral of a time pulse is approximately the pulse width (3 dB points) times the pulse peak.

$$U_0 = \frac{\theta_1 \theta_2}{4\pi}$$

where θ_1 and θ_2 are the 3-dB beamwidths, in radians, of the principal plane patterns.

$$\text{Directivity} = \frac{4\pi}{\theta_1 \theta_2} \text{ (radians) } = \frac{41,253}{\theta_1 \theta_2} \text{ (degrees)} \qquad (1\text{-}19)$$

By rearranging Eq. (1-19) we obtain the (beamwidths) directivity product.

$$\theta_1 \theta_2 Directivity = 41,253$$

Kraus uses the beamwidth to estimate the power radiated. In an aperture we use the fields in the aperture plane to determine radiated power. It is a similar operation which can also lead to other estimations of directivity from beamwidths and a (beamwidths) directivity product.

Consider a rectangular aperture. By using the approximation $u = \sin u$ for small angles in the uniform aperture distribution, the half-power beamwidth can be estimated (see Section 2.2 or Section 4.2) as

$$\text{HPBW} = 50.76° \frac{\lambda}{a}$$

Note that we have ignored the $(1+\cos\theta)/2$ pattern of the Huygens source, which reduces the beamwidth for radiation from small apertures.

We can use the result above to estimate the directivity of a rectangular aperture from beamwidths. The direcitivity of a uniform distribution is

$$Directivity = \frac{4\pi ab}{\lambda^2}$$

We solve for the aperture dimension divided by wavelength.

$$\frac{a}{\lambda} = \frac{50.76^\circ}{HPBW}$$

By substituting in the directivity formula and designating the beamwidths by θ_1 and θ_2 in the principal planes, we obtain a formula for directivity given beamwidths.

$$Directivity = \frac{32,375}{\theta_1 \theta_2}$$

This equation is reduced to a nomograph.

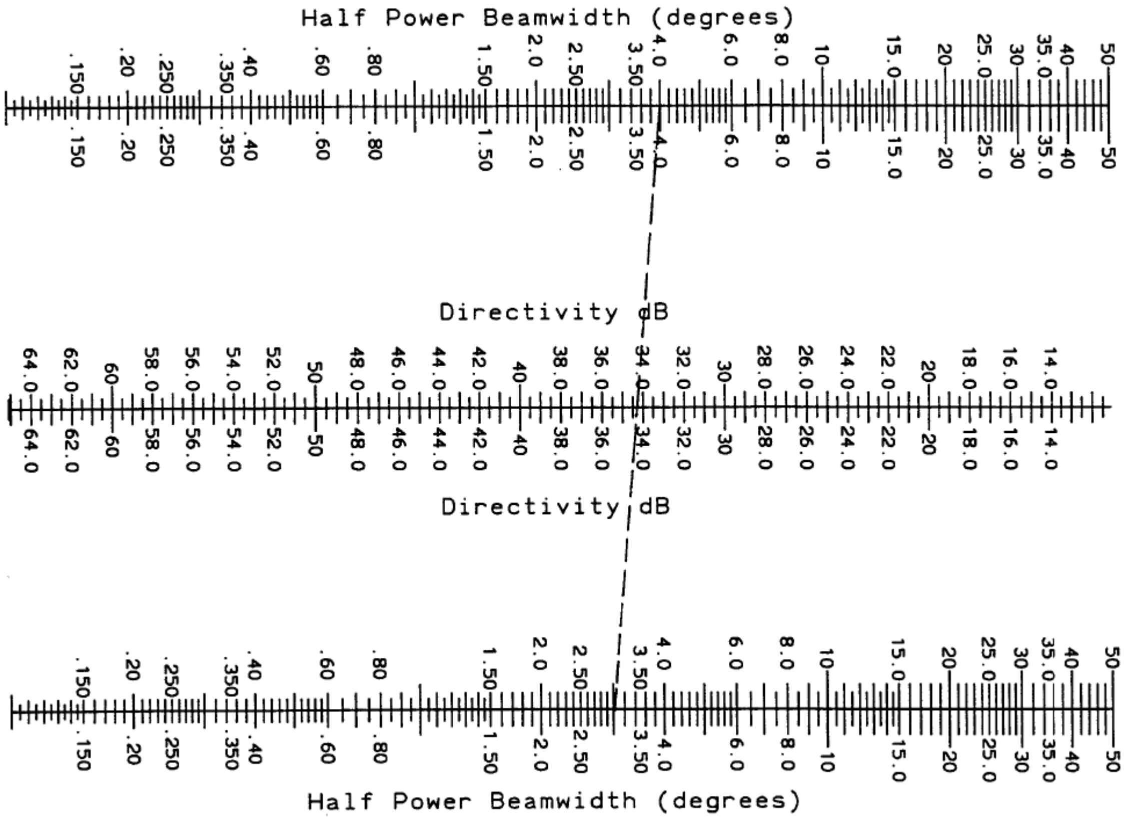

Figure 4-26-1 Directivity given *E*- and *H*-plane beamwidths

Although it has been derived for an aperture with a uniform distribution, it can be used with other distributions for an approximation. This differs from the Kraus estimate found by considering all power to be within the 3-dB beamwidth, [Eq. (1-19)] by 1 dB.

We can use amplitude taper efficiency of linear apertures to compute similar formulas. Starting with the equation from Section 4.1

$$\text{Directivity} = \frac{4\pi}{\lambda^2}\frac{\left(\iint_s |E|\,ds\right)^2}{\iint_s |E|^2\,ds} = \frac{4\pi A}{\lambda^2}\text{ATL}$$

We obtain the (beamwidths) directivity product for a rectangular separable distribution from ATL and *HPBW factor* in the two planes.

$$\theta_1\theta_2 Directivity = 50.76^2(HPBWfactor)_x(HPBWfactor)_y ATL_x ATL_y 4\pi$$

When we use a Taylor one-parameter distribution for 30 dB sidelobes in both planes of a rectangular aperture, ATL = -0.96 dB (0.8017 ratio) and *HPBW factor* = 1.355 which gives a

(beamwidths) directivity product = $4\pi\,(50.76{*}0.8017{*}1.355)^2$ = 38,206

The beamwidth equation for a uniform circular distribution of diameter *D* (see Section 4.16) is

$$\frac{D}{\lambda} = \frac{58.95}{HPBW}$$

The uniform circular aperture has the directivity.

$$Directivity = \left(\frac{\pi D}{\lambda}\right)^2$$

When we gather terms, we compute directivity by using the beamwidths θ_1 and θ_2 in the principal planes.

$$Directivity = \frac{34,300}{\theta_1 \theta_2}$$

This directivity differs by 0.25 dB from the one derived from a rectangular aperture. If we know the aperture distribution, we will use aperture efficiencies to determine directivity.

If a circular aperture has a circularly symmetrical distribution, we use Eq. (4-8) to compute ATL. Using a tapered distribution increases the beamwidth by a multiple of the uniform distribution beamwidth called the *HPBW factor*.

For a circular aperture we derive the (beamwidths) directivity product.

$$\theta_1 \theta_2 Directivity = [58.95\pi(HPBWfactor)]^2\, ATL$$

For example, a 30-dB Hansen circular distribution has a *HPBW factor* = 1.2252 and ATL = 1.19 dB.

$$Directivity = \left(\frac{\pi D}{\lambda}\right)^2 ATL = \left(\frac{\pi D}{\lambda}\right)^2 0.7603$$

$$\frac{D}{\lambda} = \frac{(HPBF\ \ factor)58.95}{\theta_1} = \frac{1.2252(58.95)}{\theta_1}$$

$$Directivity = \frac{\left(1.2252[58.95]\pi\right)^2}{\theta_1 \theta_2} ATL = \frac{39,145}{\theta_1 \theta_2}$$

A 20-dB Hansen circular distribution has *HPBWfactor* = 1.0484 and ATL = 0.09 dB. The (beamwidths) directivity product computes to the factor: 36,925. This closer to the uniform circular distribution factor of 34,300 because more of the radiated power is contained in the sidelobes which was not added to the integral of the total power. Another example of this is the 30-dB n = 8 circular Taylor distribution whose sidelobes fall off more slowly and the sidelobe region contains more power. This distribution has *HPBWfactor* = 1.1079 and ATL = 0.54 dB which computes to a (beamwidths) directivity product = 37,176

Each aperture distribution produces a similar (beamwidths) directivity product with similar values in the range from 32,375 to 39,145 or a maximum variation of about 0.8 dB. Considering measurement of gain has a accuracy error greater than 0.5 dB, measurement is unable to decide which factor should be used. The EIA-411 standard uses 31,000 for a ground station which could account for the spillover loss of the feed antenna pattern [see Section 8-2].