### 9-3.3 Ray tracing with matching layers on lens

To reduce internal reflections in a lens, quarterwave layers or multiple step layers of gradually reduced dielectric constant are placed on the outer surfaces of lens. We form the tapered layers by cutting grooves in the lens using a thread cutting tool or a special ground tool to reduce the sharp edges at their bases and tops. We divide the tapered layers into many constant dielectric layers and consider the refraction at the surfaces between layers and the final layer and free space.

We start with Snell's law with a ray approaching from the inner layer at an angle $\theta_2$ where the medium has an index of refraction of $n_2$ and the ray is bent at the planar surface to an angle $\theta_1$ in the exit medium with index of refraction $n_1$. We rearrange Eq. (2-68) to compute the exit ray direction relative to the surface normal.

$$n_2 \sin\theta_2 = n_1 \sin\theta_1 \text{ or rearranging } \sin\theta_1 = \frac{n_2}{n_1}\sin\theta_2$$

We can extend this equation to a multilayer dielectric slab matching structure on the lens. With a ray inside the lens approaching the planner surface at an angle $\theta_N$ to relative to the normal in a medium index of refraction $n_N$ and an external medium $n_0$, the external ray direction is found.

$$\sin\theta_0 = \frac{n_N}{n_{N-1}}\frac{n_{N-1}}{n_{N-2}}\cdots\frac{n_2}{n_1}\frac{n_1}{n_0}\sin\theta_N$$

We can reduce this equation by canceling matching terms.

$$\sin\theta_0 = \frac{n_N}{n_0}\sin\theta_N$$

The exciting direction is independent of the index of refractions of the intermediate layers. The matching layers have no affect on the lens exciting ray directions and do not change the lens operation. Of course, the layers cause a lateral shift of the rays due their thickness.

If the rays approach the internal surface at a large enough angle that $\sin\theta_0 > 1$, the rays are totally reflected. Applying matching layers cannot make the totally reflected ray exit the lens because the exit direction is determined by the ratio of the initial and final layer index of refractions.