FREQUENCY INDEPENDENT ANTENNAS

The frequency independent antennas are based on the principle of frequency scaling used in model measurements. The pattern and polarization of an antenna do not change if the dimensions are scaled with the same factor as the wavelength. The losses do not scale properly but the conductivity of the metals that antennas are made of is very high and the dielectric loss tangents are small. A change of scale is permissible with small losses. Of course, scaling an antenna to a higher frequency presents problems in fabrication since it is difficult to scale tolerances, and wall thicknesses become too thin.

In order to have a frequency independent antenna we need structures which can be their own scale models. One solution is to have an antenna which is specified only in terms of angles (Rumsey 1955). This solution leads to the class of spiral antennas which are continuously scaled structures. The second solution is to include in the antenna scale model parts for which the antenna is scaled exactly. Between these frequencies the antenna is not exactly scaled and is not frequency independent. The pattern response will ripple with frequency. The scaling on these antennas will be log periodic. We have two types of scaling for frequency independent antennas: continuous and log periodic. This is only the first requirement for a successful antenna.

The structures given above have no ends. They must scale down to an infinitesimal size and grow without bound. Note that even an infinite log periodically scaled antenna will have a frequency ripple between scalings. A practical antenna must have a finite size which gives us another requirement on the antenna. The current must decay before the end of the antenna is reached or the reflected wave will change the pattern shape and polarization. The antennas are usually fed from the high frequency or small end. This portion of the antenna which does not radiate must be a transmission line for the low frequency currents before the active or radiating region is reached.

The design of these antennas is determined in two parts. The pattern characteristics determine the scaling constant of log periodic antennas or the wrap rate of the spirals. The actual size is determined by upper and lower truncation constants which determine how far the structure must be continued beyond the resonant portions of the antenna until the currents have been reduced sufficiently to be able to truncate the structure without degrading the pattern significantly. The truncation constants are nebulous since some applications can use antennas with a large amount of pattern distortion.

A frequency independent antenna has the same beamwidth and directivity when the frequency is increased. Usually there is a rotation of the beamwidth angle with a change in frequency as in a spiral antenna, but once this is accounted for the antenna is frequency independent. Most antennas, like horns, have increasing directivity and decreasing beamwidths with increasing frequency. This is because the physical aperture remains constant and the wavelength decreases. The gain (directivity) is related to the aperture by

$$g = \frac{4\pi A}{\lambda^2}$$

But for structures defined only by angles, this is not the case. The directivity remains constant with increasing frequency. This can happen only if the effective area decreases as the square of the wavelength as the frequency is increased. The currents on the structure must be attenuated rapidly after the active region is reached and the area of the active region must be proportional to the square of wavelength. Antennas defined by angles will satisfy the truncation requirement for a practical frequency independent structure since the currents are highly attenuated after an active region which shrinks with increasing frequency. Any antenna which will satisfy the frequency independence of its beamwidths and directivity must have a limited active region followed by an attenuated current region beyond the active region.

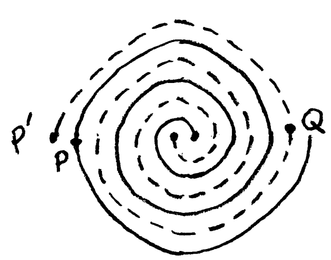
The pattern is also effected by the truncation requirement. The pattern is proportional to the magnetic field. If the magnetic field was radiated in the direction of the antenna structure, then the tangential magnetic field would excite currents on the structure. This would not satisfy the truncation principle because there would be currents on the structure beyond the active region. The pattern will be zero in the directions of the infinite structure for increasing dimensions. In the case of a planer spiral the fields are zero in the direction of the spiral if it is frequency independent (defined by angles only). The conical spiral must radiate towards the top of the cone to satisfy the truncation requirement. It is a backfire antenna. Similiarly the log periodic dipole antenna will be a backfire antenna and radiate in the direction of the small end of the antenna.

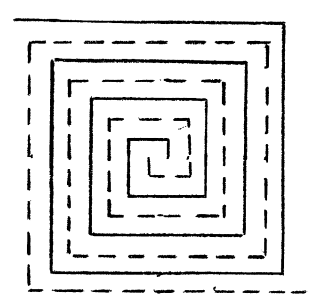
We will approach these antennas in an historical order. The equiangular spiral, a true frequency independent antenna, was an out growth of the Archimedes spiral which is broadband but not frequency independent. We will cover this archimedian spiral first.

The Archimedes spiral was developed at Wright Air Development Center by E. M. Turner during 1953 and 1954. This antenna is not a true frequency independent antenna since it does not meet the truncation requirement. The currents do not decrease sufficiently after the active region so that the spiral may be truncated without effecting the pattern response. An archemedian spiral increases uniformly with increasing angle.

$$r = r_o + a\,\phi$$

Where r_o is the initial radius and a is the growth rate. This antenna does not scale to an infinitesimal size which is one of the requirements of a frequency independent antenna. Since the current does not decrease sufficiently after the active region, the last few turns must be loaded to prevent reflections. This antenna is made as circular spirals or square spirals as shown below.





The second arm of the spiral is shown dashed so that it is easier to distinquish between the windings. It too is a solid line of copper. The antenna is usually made complementary; the width of the line equals the spacings between the lines. From this we know that the input impedance will be close to the impedance of an infinite complementary structure: 189 ohms.

The center of the spiral is fed from a balanced line. When the spiral starts, the currents on the turns are nearly equal and opposite. The combination of the fields from these currents will cancel in the far field like the fields from a two wire transmission line. The currents are separated as the spiral grows from the feed. When the circumference of the turns approach one wavelength, then the currents which are out of phase at P and Q in the diagram above become in phase at points P and P'. The current travels around half the turn from Q to P' and decreases in phase from 180° to 0°. Now the currents are in phase and the far field from the currents will add. It appears that the radiation is from a traveling wave on a loop one wavelength in circumference.

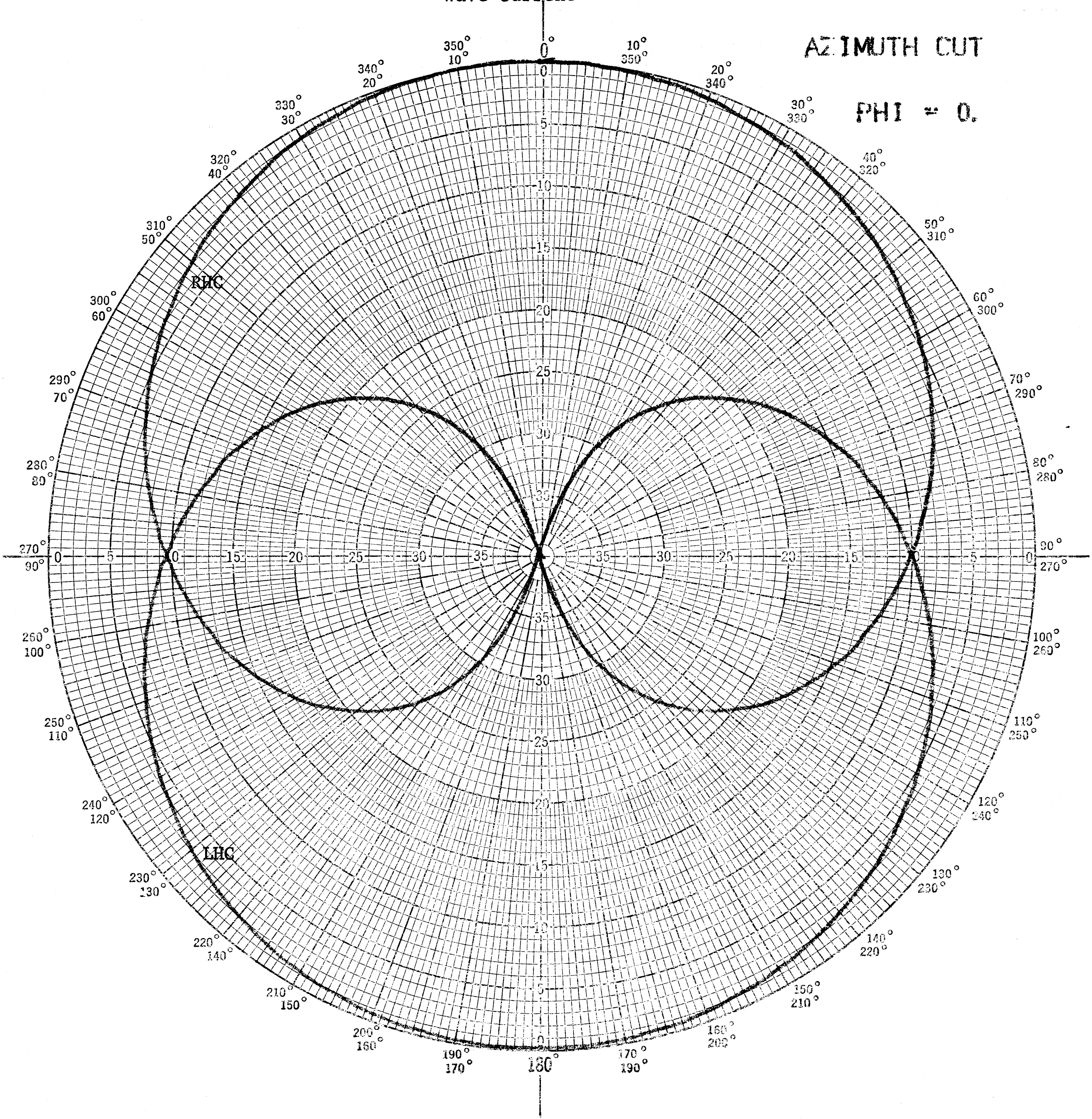
The spiral is similiar to the axial mode helical antenna which is about one wavelength in circumference. The traveling wave will produce a right hand circular radiation above the plane of the spiral as shown. The spiral radiates a left hand circular polarized wave below. From symmetry we can see that the pattern shape below the spiral will be the same as above, unlike the helical antenna which has a progressive wave traveling in the Z direction. The helical antenna has a pattern for a single turn on page 422 which has opposite sense polarization on the two sides but a 4 dB front to back ratio. On page 437 is a pattern of a flat loop one wavelength in circumference with a traveling wave on it. The patterns of the two polarizations are equal and opposite. This is only approximately the pattern of the spiral because the spiral increasing has been ignored and the effect from nearby turns has been ignored.

The current on the flat spiral can be described by using fundamental modes as was done for the helical antenna. The general mode will be

$$e^{\pm jm\phi}$$

where m is an integer. The radiation from the spiral is the m = 1 mode (T_1) which is excited on the turns which are near one wavelength in circumference. On the turns near the feed, the currents are in the T_0 mode which is a transmission line mode that couples into the next higher mode as the diameter of the spiral increases. In the analysis of the spiral as a transmission line given above the effects of mutual coupling between the turns was ignored. The coupling will have the effect of changing the velocity of the currents on the windings. A group of windings near one wavelength in circumference will all be excited in the T_1 mode where the velocity on the wires will adjust to the circumference. This is similiar to the effect on the helical antenna. When the spiral continues to larger diameters, the energy is coupled into higher order modes (T_2 and T_3). These modes are highly attenuated and tend to cancel each other. Because the spiral increases uniformly, the cancellation is quite slow and there will still be significant

One Wavelength in Circumference Loop with a Traveling Wave Current



Polar Chart No. 127D
SCIENTIFIC-ATLANTA, INC.
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currents when the end of the spiral is reached. If the end turns are not loaded then the energy is reflected and travels back in towards the center of the spiral. When the currents reach the portion of the spiral near one wavelength in circumference, the energy is radiated from the T_1 mode. This wave will be in the opposite sense of circular polarization and increases the boresight axial ratio.

The size of the antenna is determined by the lowest operating frequency. The circumference must be at least one wavelength to 1.25 wavelengths at the outer edge. Beyond that the next few turns should be terminated in a lossy material to absorb the remaining energy and retain a good axial ratio on boresight. Usually the antenna is mounted over a cavity to prevent the opposite sense polarization from radiating. If the spiral is placed a quarterwave over a ground plane, the field reflected off the ground plane will change the sense of polarization and add in phase with the directly radiated fields on the open side. The wave travels a half wavelength going from the spiral and back. Upon reflection the wave gains 180^o (a phase reversal) to satisfy the boundary conditions on the horizontally polarized wave which gives a total phase shift of 360^o.

The antenna must be fed from a balun or the in phase component of the unbalanced mode will reach a point on the spiral where the fields from the currents on the two windings will add in phase (two wavelengths in circumference) and squint the beam. The in-phase component will also radiate from the first few turns. Most of the problems with the antenna is finding a suitable balun and phasing the reflections from the cavity to achieve good axial ratio response.

EQUIANGULAR SPIRAL

Rumsey has argued that if the shape of an antenna is determined entirely by angles, its performance will be independent of frequency because it will be invariant to a change of scale. There is no specified length. One example he gives is the biconical antenna which is specified only by angles but it fails to satisfy the truncation principle. The antenna must be infinitely long to be truely frequency independent. The current fails to decrease along the length of the antenna. The most likely antenna is the equiangular spiral which is specified only be angles.

The equiangular spiral was the first practical frequency independent antenna. The spiral is drawn on page 439. It is defined by the equation

$$r = r_o \, e^{a\phi}$$

The constant r_o can be related to an angle by

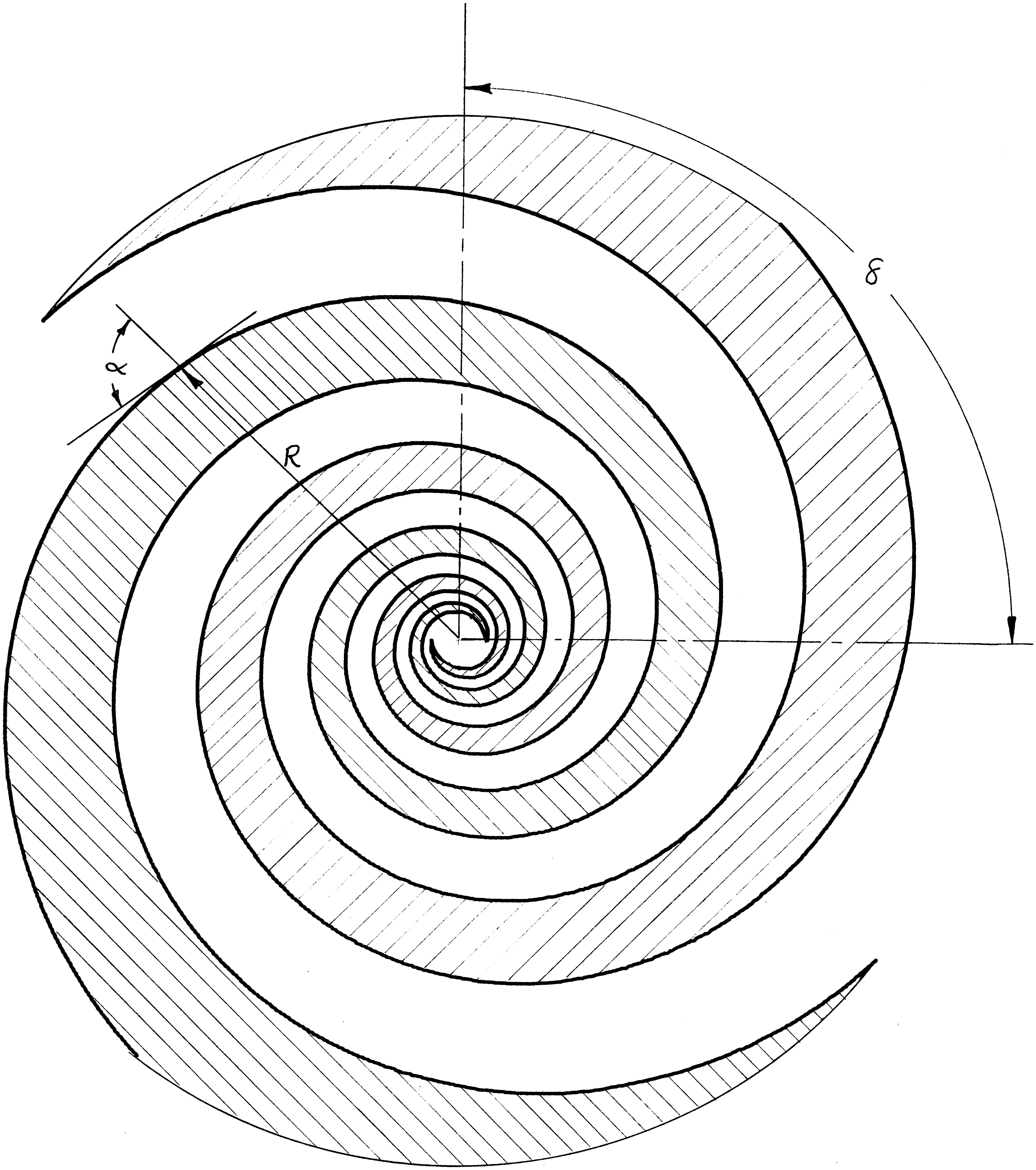
$$r_o = e^{a\phi_o}$$

The constant a is related to the angle α, the wrap angle, as shown on the diagram of the spiral.

$$a = 1/ \mathrm{Tan}\, \alpha$$

Two Arm Equiangular Spiral

Complementary Conductors



The thickness of the spiral arms is also expressed in terms of angles. The spiral is rotated by an angle δ to determine the other edge of the spiral.

$$r = r_o e^{a(\phi + \delta)}$$

These two equations define one arm of the equiangular spiral. The other arm of the spiral is defined by the equations

$$r = r_o e^{a(\phi + \pi)} \qquad \text{and}$$

$$r = r_o e^{a(\phi + \pi + \delta)}$$

For the antenna shown on page 439, the angle δ is $\pi/2$ which will give a self complementary antenna. We know that the impedance will be close to 189 from the Babinet-Booker principle. A drawing of a non complementary spiral is given on page 441; this antenna will have a higher input impedance than the spiral drawn on page 439.

The practical antenna must be truncated. The length of one of the arms can be found by integrating the equation for arc length. The result is

$$L = (r - r_o)\sqrt{1 + \tan^2\alpha} = (r - r_o)\sqrt{1 + 1/a^2}$$

The antennas drawn on pages 439 and 441 will radiate a right hand circularly polarized wave above the spiral and a left hand wave below. From symmetry we can see that the patterns will be identical. This is similiar to the archimedian spiral. The approximate pattern is given on page 442. The pattern is pretty much constant over a reasonable range of parameters.

The characteristics of this antenna were measured extensively by John D. Dyson with some of the results given in the paper: "Equiangular Spiral Antenna", IEEE, AP April 1959. He found that the antenna must have at least 1½ turns in each arm before it would show reasonable patterns. The truncation was found by measuring many different antennas. On page 443 is a curve which gives the truncation constant in terms of another constant: K.

$$K = e^{-a\delta}$$

The length along the arm necessary to radiate the lowest frequency with a 3 dB axial ratio is a function of the wrap angle and the width of the windings (δ). Dyson measured spirals with wrap angles ranging from 66^o to 79^o for this curve. The upper truncation constant determines the start radius necessary to achieve good patterns at the highest frequency. The initial radius should be at most $\lambda/8$.

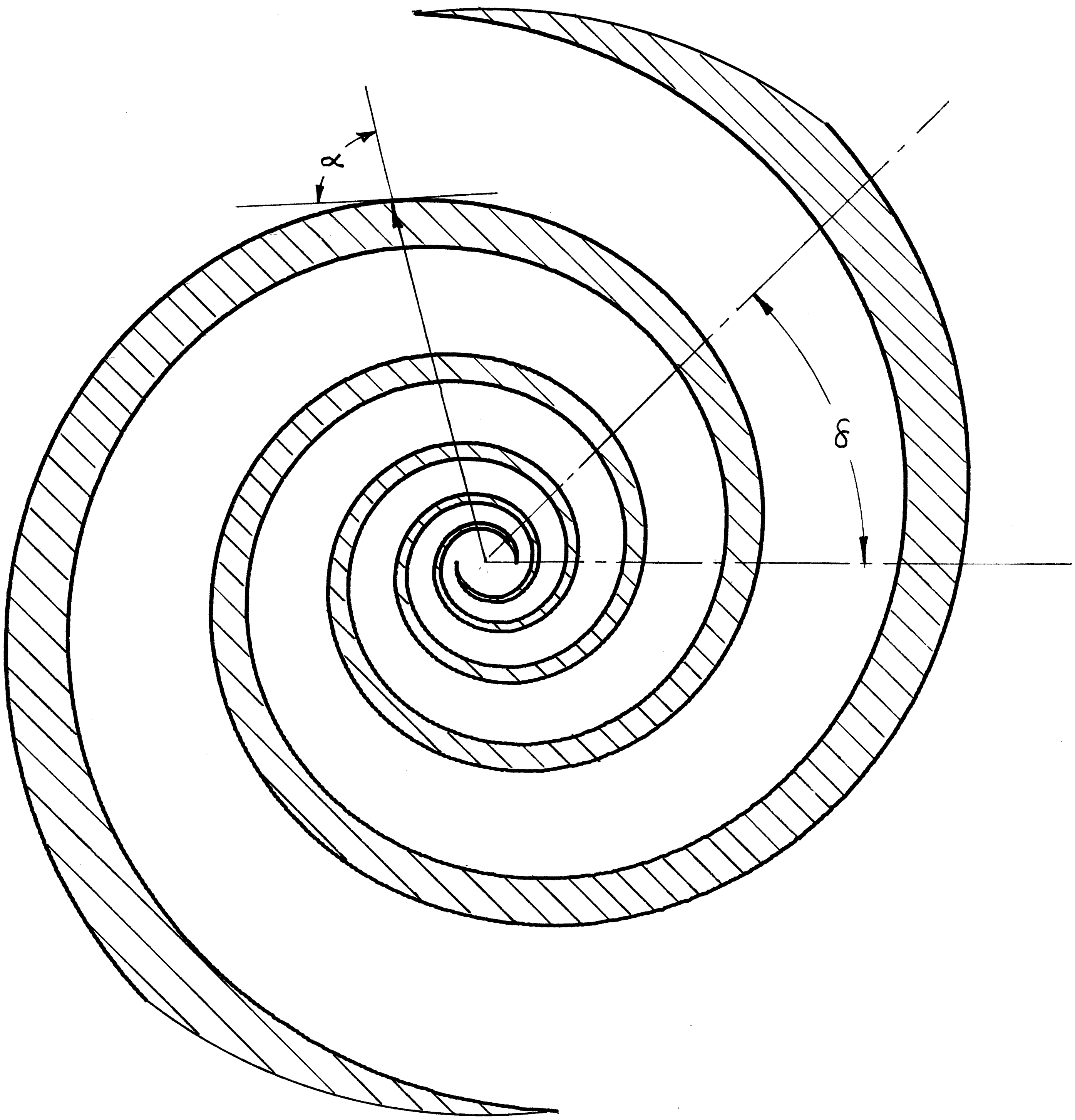
Example: Design an equianular spiral from 1 GHz to 12 GHz with a wrap angle of 70^o and self complementary arms.

The initial radius is determined by the highest frequency.

$$r_o = \lambda/8 \cong 0.12 \text{ inches}$$

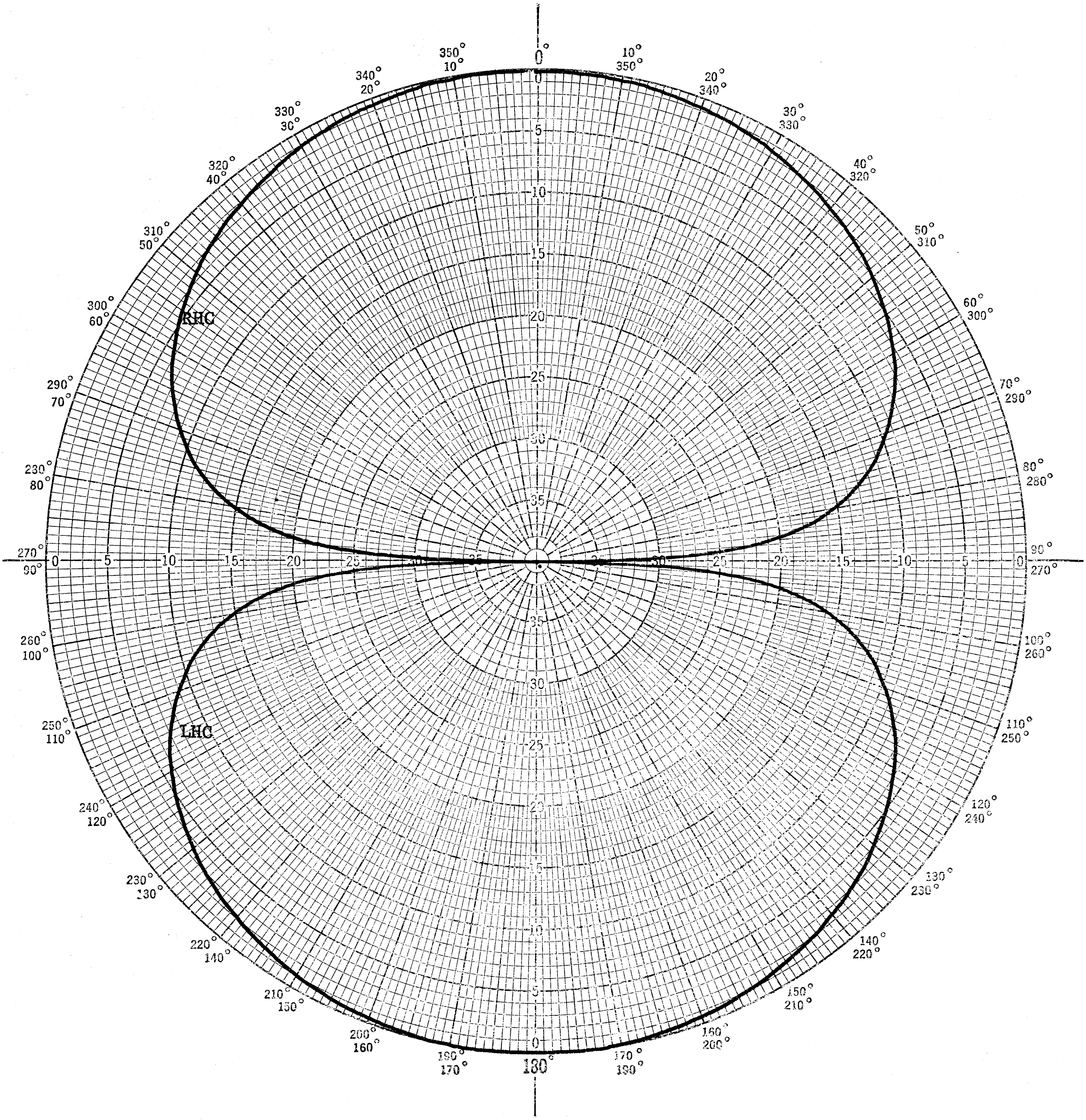
Two Arm Equiangular Spiral

Non Complementary Conductors



Approximate Pattern

Two Arm Equiangular Spiral



Polar Chart No. 127D
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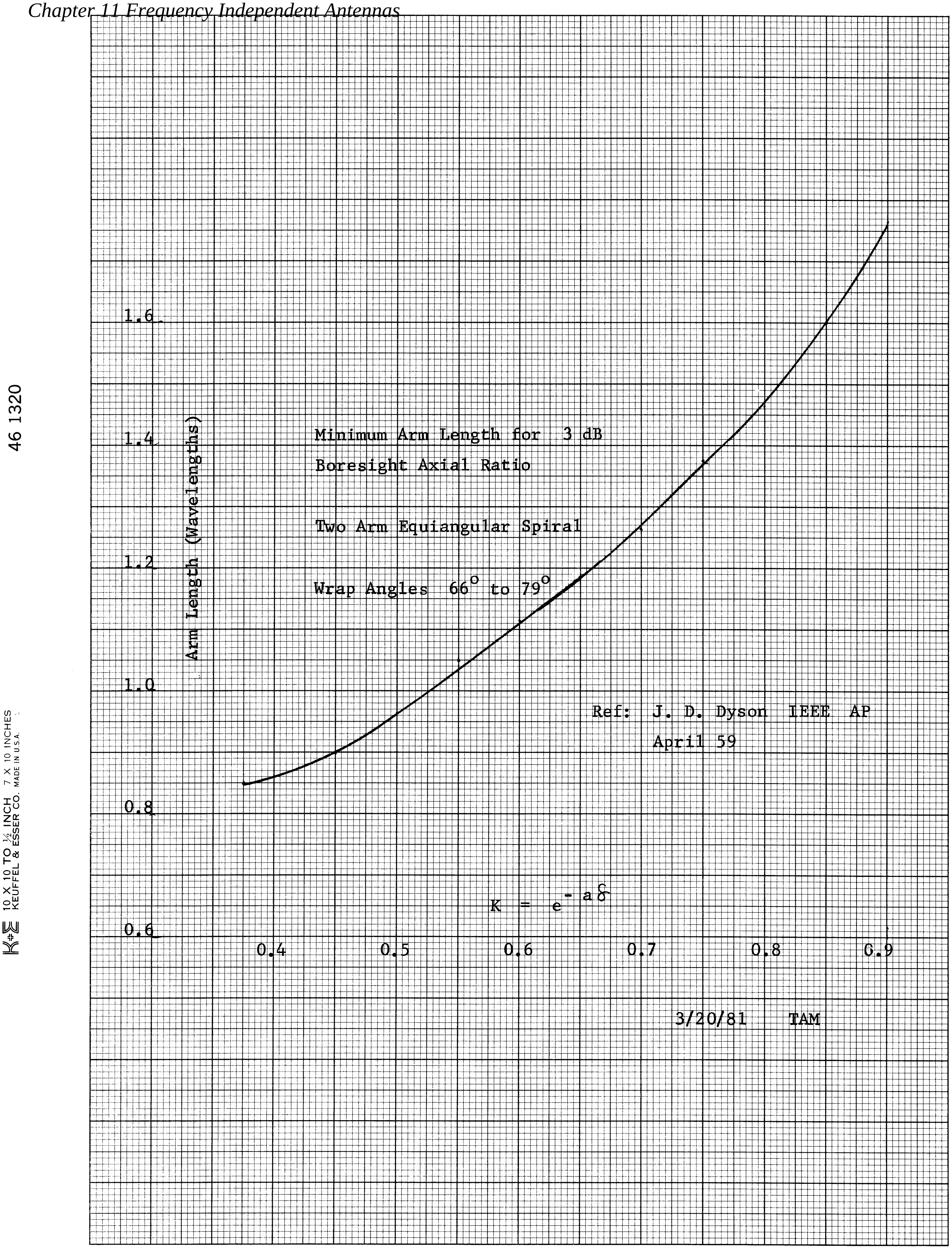

Minimum Arm Length for 3 dB
Boresight Axial Ratio
Two Arm Equiangular Spiral
Wrap Angles 66° to 79°
Arm Length (Wavelengths)
1.6
1.4
1.2
1.0
0.8
0.6
0.4
0.5
0.6
0.7
0.8
0.9
Ref: J. D. Dyson IEEE AP
April 59
K = e^-aδ
3/20/81 TAM
46 1320
10 X 10 TO ½ INCH 7 X 10 INCHES
KEUFFEL & ESSER CO. MADE IN U.S.A.

We will pick the initial radius to be 0.1 inch. We now need to find the constant K to use the curve on page 443 for the length of the spiral.

$$K = e^{-\pi/(2\tan(70^\circ))} = .56$$

From the curve the length of the spiral at the lowest frequency must be 1.05 wavelengths.

$$L = 12.4 \text{ inches}$$

This is the length along one of the edges of the spiral which are all the same if the end of the spiral is an arc about the center as shown on pages 439 and 441. The outer radius is found from the equation for length.

$$r = r_o + \frac{L}{\sqrt{1+\tan^2\alpha}}$$

$$r = 4.34 \text{ inches}$$

We can compare this to an archimedian spiral which is 1.25 wavelengths in circumference at the lowest frequency.

$$r = 2.35 \text{ inches}$$

The archimedian spiral would be continued a few more turns for the loading, but it has a much more favorable size compared to the equiangular spiral.

In this example of the equiangular spiral we can find the number of turns from the equation of the curve.

$$r = r_o e^{a\phi}$$

$$\phi = \frac{1}{a}\ln\left(\frac{r}{r_o}\right) = (\tan\alpha)\ln\left(\frac{r}{r_o}\right)$$

$$\Delta\phi = 10.36 \qquad \text{TURNS} = \frac{10.36}{2\pi} = 1.65$$

This is the number of turns in each arm. Suppose the antenna is made with a wrap angle of 80°. The inner radius would still be 0.1 inches. The factor K is calculated:

$$K = e^{-\pi/(2\tan(80^\circ))} = .276$$

This point is off the graph given on page 443. We can see from the graph that an arm length of one wavelength would be sufficient.

$$L = 11.8 \text{ inches}$$

The outer radius is given as $$r = 0.1 + \frac{11.8}{\sqrt{1+\tan^2(80^\circ)}} = 2.15$$

This compares well with the archimedian spiral. It has 2.77 turns in each arm. In order to have the smallest possible diameter we need to design with large wrap angles.

The beamwidths of a great circle pattern will changes slightly as the frequency changes. This is especially true for small wrap angles (large a). With the same antenna the beamwidth changes for various patterns. The pattern with a given beamwidth will rotate with frequency with respect to the antenna. This comes from the fact that the frequency scaling is achieved by rotating the active region on the arms. The scaling is proportional to the radius. If the frequency increases from f_1 to f_2, this is proportional to an angle rotation.

$$f_2/f_1 = r_2/r_1 = e^{a(\phi_2 - \phi_1)}$$

$$\Delta\phi = \frac{1}{a}\ln(f_2/f_1)$$

The rotation is proportional to the logarithm of frequency. The equiangular spiral is also a log periodic structure in this sense. This spiral is also called a log spiral.

Feeding the Equiangular Spiral

The spiral must be fed with a balanced line or the even mode (unbalanced mode) radiation will squint the beam. The properties of the antenna can be used to form a balun. The truncation condition means that the current attenuates rapidly beyond the active region. This means that currents which are induced on the antenna outside the active region do not get coupled into the input terminals. This property is used to make the balun. A coax cable is soldered to one of the arms with a connector beyond the large end of the spiral. At the center of the spiral a connection is made from the center conductor of the coax to the other arm. To maintain symmetry an outer shield of a coax is soldered on the other arm. The current on the outside conductor flows from inside the conductor to the outside of the conductor and on to the arms of the spiral. Any currents which are induced on the outer shield of the coax beyond the active region will not reach the connection because the outer shield is now part of the antenna which satisfies the truncation requirement. This is called an infinite balun. See page 446 for a diagram of the connection.

The spiral may also be mounted over a cavity so that the opposite sense polarization will not radiate. The antenna may be fed from a balun like the archimedian spiral when mounted in the cavity.

Like the helical antenna and the archimedian spiral, the equiangular or log spiral currents can be expanded in terms of the fundamental modes:

$$e^{\pm jm\phi}$$

Only the terms $m = \pm 1$ give a contribution to the field on boresight and correspond to the left and right hand circular polarization components. Rumsey in Frequency Independent Antennas, 1966 shows that the current on the arms does not follow the direction of the windings but varies in direction throughout the spiral with the amplitude of the current decreasing rapidly after the active region. As the frequency changes the direction of the current at a particular point on the spiral changes direction.

CONICAL LOG SPIRAL ANTENNA

The planar equiangular spiral has the following problems. It radiates equally on both sides with opposite polarizations. This can be fixed by mounting the antenna over a cavity. The second problem is that the beamwidth cannot be varied. These problems can be solved by forming the equiangular spiral on a cone. The high frequency portion is near the apex of the cone. Because of the truncation principle which the spiral continues to satisfy when bent into a cone, the radiation tends to zero in the direction of the increasing cone. This leaves the antenna as a backfire antenna with the maximum radiation in the direction of the feed. The antenna must be a fast wave structure in the active region for this to occur. Of course, this antenna is a fast wave structure in the axial direction. This is in constrast with the helical antenna which is a slow wave structure and a forward fire antenna. The result is the opposite of the helical antenna; the cross polarization circular component in the direction of the traveling wave current decreases while the backfire component increases.

A figure of the conical spiral antenna is drawn on page 448. The arms of the antenna are projected on a cone defined by the angle θ_o from the center line of the cone to the cone surface. The equation of each spiral on the cone is given by

$$\rho = \rho_o e^{b\phi} \qquad b = \frac{\sin\theta_o}{\tan\alpha}$$

The angle α is the angle of the spiral with respect to the line drawn from the vertex to the windings along the cone. ρ is the distance from the vertex to the spiral. Note that this reduces to the planar spiral if $\theta_o = \pi/2$. The width of the windings is defined by the angle δ shown in the figure. The length of the windings is given by

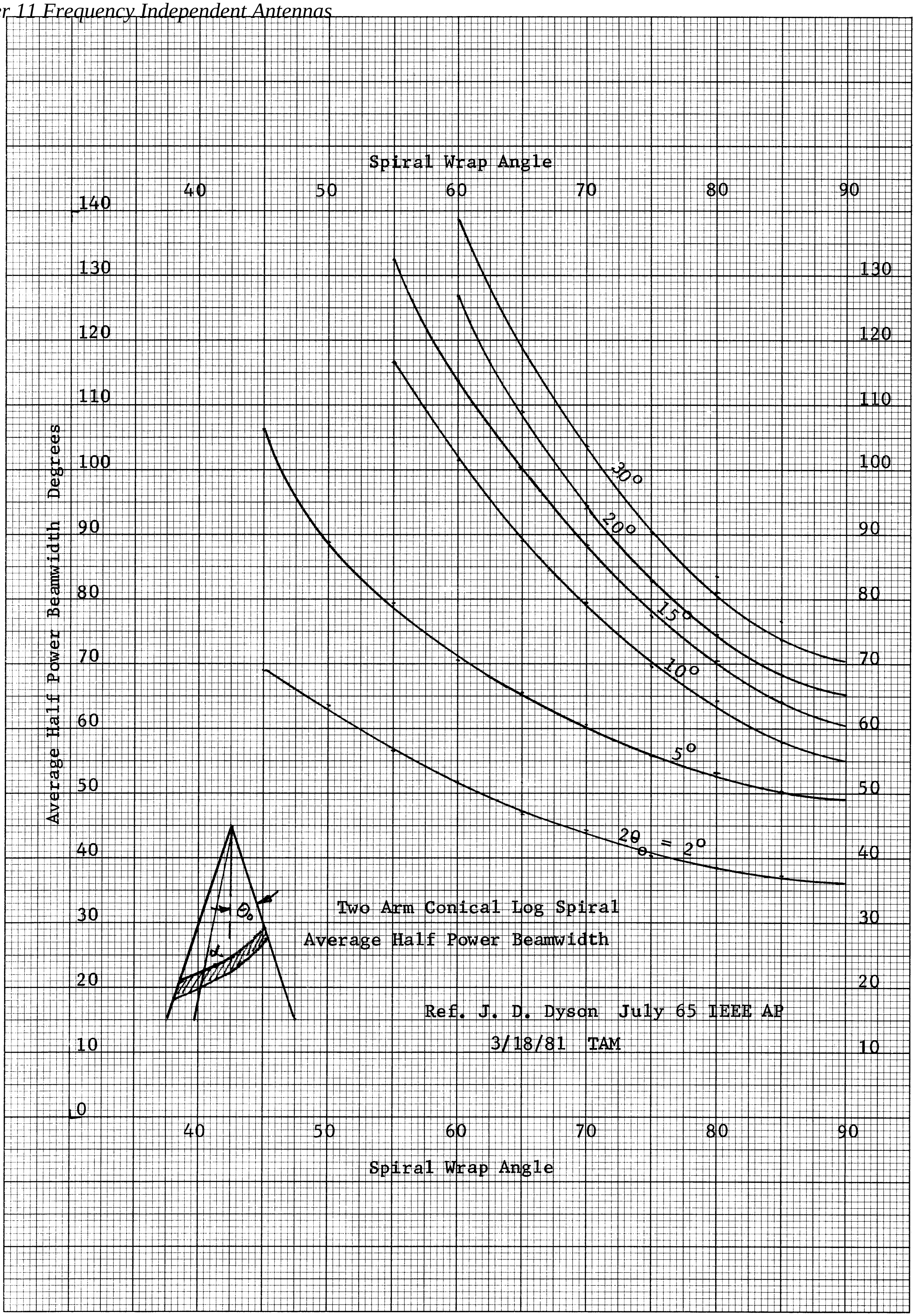
$$L = (\rho - \rho_o)\sqrt{1 + 1/b^2} = (\rho - \rho_o)\sqrt{1 + \tan^2\alpha/\sin^2\theta_o}$$

which is similiar to the planar spiral.

The characteristics of this antenna were also measured by J. D. Dyson[1]. As with the planar spiral he made a large number of antennas and measured their characteristics. The pattern is predominately unidirectional toward the tip of the cone. The average half power beamwidth can be related to the cone and wrap angles. This has been plotted on page 449. Using these beamwidths we can estimate the directivity from the nomograph on page 38 with a horizontal line. In general the directivity increases for increasing wrap angles and decreasing cone angles. Note that the beamwidth can be varied only over a limited range.

1. Dyson, J. D., "The Unidirectional Equiangular Spiral Antenna", IEEE Trans. AP, Oct. 1959.

Dyson, J. D., "The Characteristics and Design of the Conical Log-Spiral Antenna", IEEE Trans. AP, July 1965.


Spiral Wrap Angle
40
50
60
70
80
90
140
130
120
110
100
90
80
70
60
50
40
30
20
10
0
Average Half Power Beamwidth Degrees
30°
20°
15°
10°
5°
2θ₀ = 2°
θ₀
α
Two Arm Conical Log Spiral
Average Half Power Beamwidth
Ref. J. D. Dyson July 65 IEEE AP
3/18/81 TAM
Spiral Wrap Angle

The necessary size of the antenna can be determined from near field measurements which were correlated to the far field patterns. If a small probe is positioned along the windings on a radial line from the vertex, the current on the windings is found to increase and peak in the active region. After the active region, the current attenuates rapidly. The pattern is not changed if the current has decreased at least 3 dB from the peak current on the small end. This establishes the upper truncation constant. Similiarly, if the lower diameter is sufficiently large enough so that the current is attenuated by 15 dB, there will be no change in the pattern if the antenna is truncated. On page 451 is a plot which gives the upper and lower truncation constants for various design parameters.

Example. Design a conical log spiral with a cone angle of 15^o with a wrap angle of 80^o from 1 GHz to 3 GHz.

$$2\,\theta_o = 30^o$$

$$\text{Upper truncation constant,}\quad a_3^-/\lambda = .07$$

$$\text{Lower truncation constant,}\quad a_{15}^+/\lambda = .22$$

These are the radii of the cone at the top and bottom. The upper cone diameter is determined by the upper frequency.

$$\text{Upper Diameter} = (2)(.07)(11.80285/3) = 0.55 \text{ inches}$$

The lower diameter is determined by the lower frequency and the lower truncation constant.

$$\text{Lower Diameter} = (2)(.22)(11.80285) = 5.193 \text{ inches}$$

The height is found from the diameters and the cone angle.

$$\text{Height} = (5.193 - .55)/(2 \operatorname{Tan} \theta_o) = 8.664 \text{ inches}$$

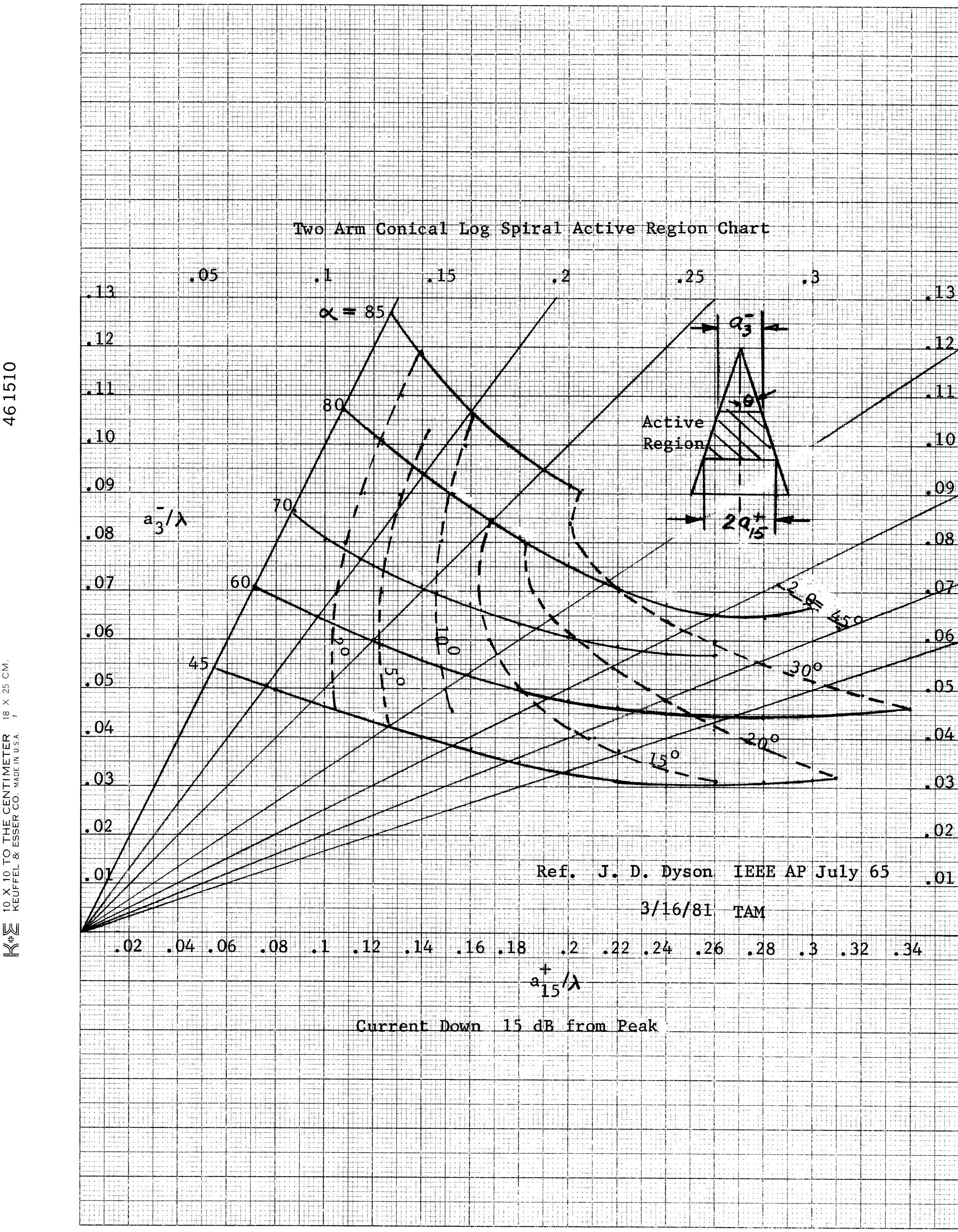
The antenna can be designed somewhat smaller if the pattern is allowed to degrade at the band limits. A curve for this design is given on page 452. The lower truncation constant now corresponds to 10 dB attenuation of the current from the peak in the active region. The example can be recalculated.

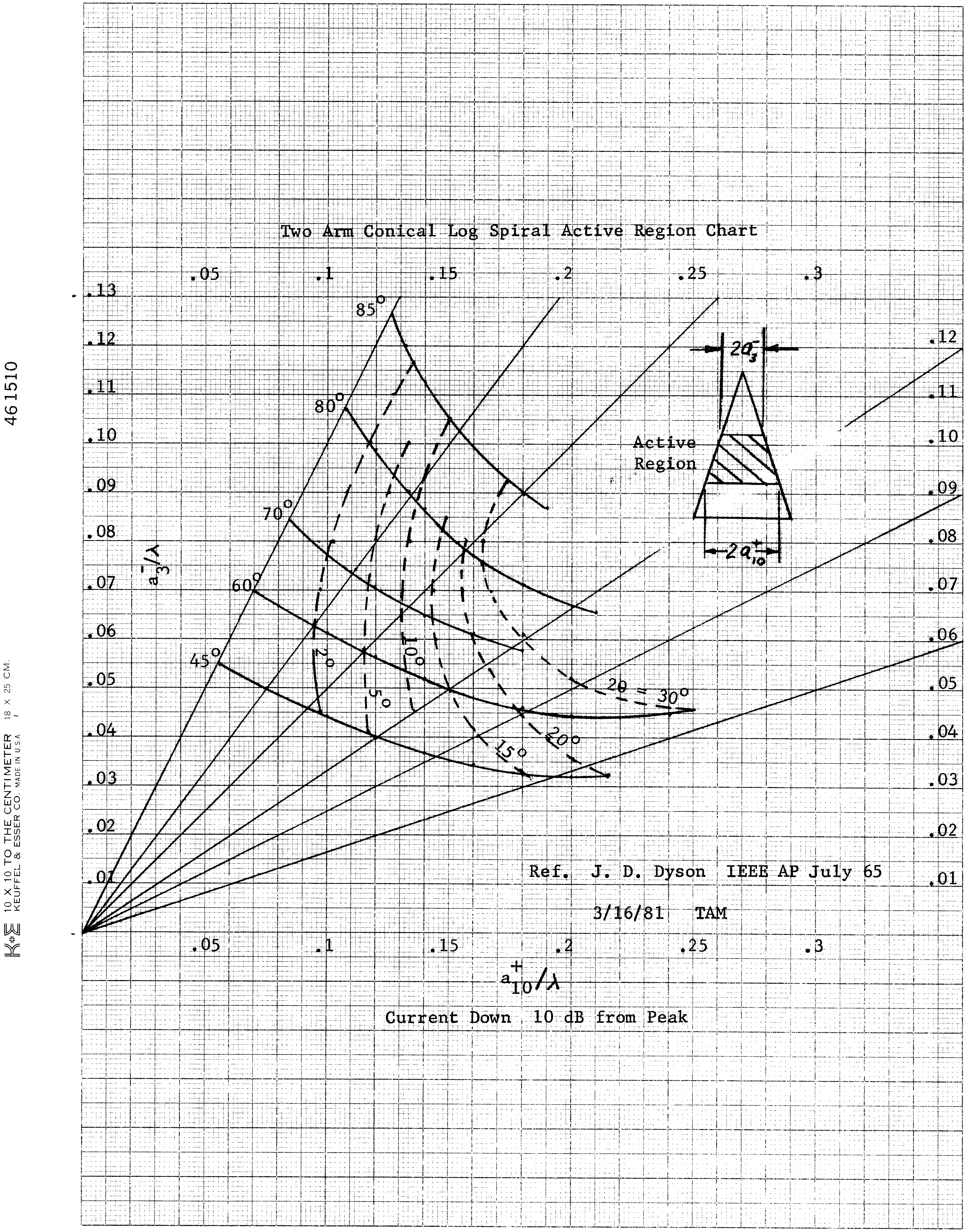
$$\text{Upper Diameter} = (2)(.076)(11.80285/3) = 0.598 \text{ inches}$$

$$\text{Lower Diameter} = (2)(.164)(11.80285) = 3.871 \text{ inches}$$

$$\text{Height} = 6.108 \text{ inches.}$$

The polarization of the antenna can be determined by viewing the antenna from the vertex. It is determined by the hand rule with the thumb in the direction of propagation.


Two Arm Conical Log Spiral Active Region Chart
.05
.1
.15
.2
.25
.3
.13
.12
.11
.10
.09
.08
.07
.06
.05
.04
.03
.02
.01
α = 85
80
70
60
45
a_3^-/λ
Active Region
θ
a_3^-
$2a_{15}^+$
2°
5°
10°
15°
20°
30°
45°
Ref. J. D. Dyson IEEE AP July 65
3/16/81 TAM
.02
.04
.06
.08
.1
.12
.14
.16
.18
.2
.22
.24
.26
.28
.3
.32
.34
a_{15}^+/λ
Current Down 15 dB from Peak
461510
10 X 10 TO THE CENTIMETER 18 X 25 CM.
KEUFFEL & ESSER CO. MADE IN U.S.A.


Two Arm Conical Log Spiral Active Region Chart
.05
.1
.15
.2
.25
.3
.13
.12
.11
.10
.09
.08
.07
.06
.05
.04
.03
.02
.01
85°
80°
70°
60°
45°
2°
5°
10°
15°
20°
2θ = 30°
a_3^-/λ
$2a_3^-$
Active Region
$2a_{10}^+$
Ref. J. D. Dyson IEEE AP July 65
3/16/81 TAM
a_{10}^+/λ
Current Down 10 dB from Peak
46 1510
10 X 10 TO THE CENTIMETER 18 X 25 CM.
KEUFFEL & ESSER CO. MADE IN U.S.A.

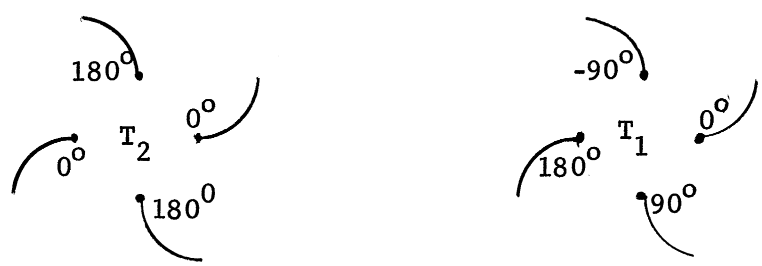
This antenns can also be analyzed by the same mode theory as the helical antenna and the planar spiral. The usual radiation mode described above is the T_1 mode. It is the only mode with its radiation peak on boresight. We can understand the attenuation of the current on the spiral as being due to the coupling of the lower order modes into the higher order modes as the diameter increases. Rumsey has suggested that this coupling is due the the curvature of the windings.

MODE 2 CONICAL LOG SPIRAL ANTENNA

There are applications where it is desireable to have a butterfly pattern and still have circular polarization at the beam peak. The first higher order radiating mode (T_2) of the conical log spiral will give such a pattern. The easiest way to achieve this mode is to use four arms in the spiral. This mode requires that the phase change 4π as the antenna is rotated once around or π for 90° of rotation. The mode function is

$$e^{-j2\phi}$$

for a right hand circularly polarized wave. The feed for the 4 arm spiral is given as the diagram below.



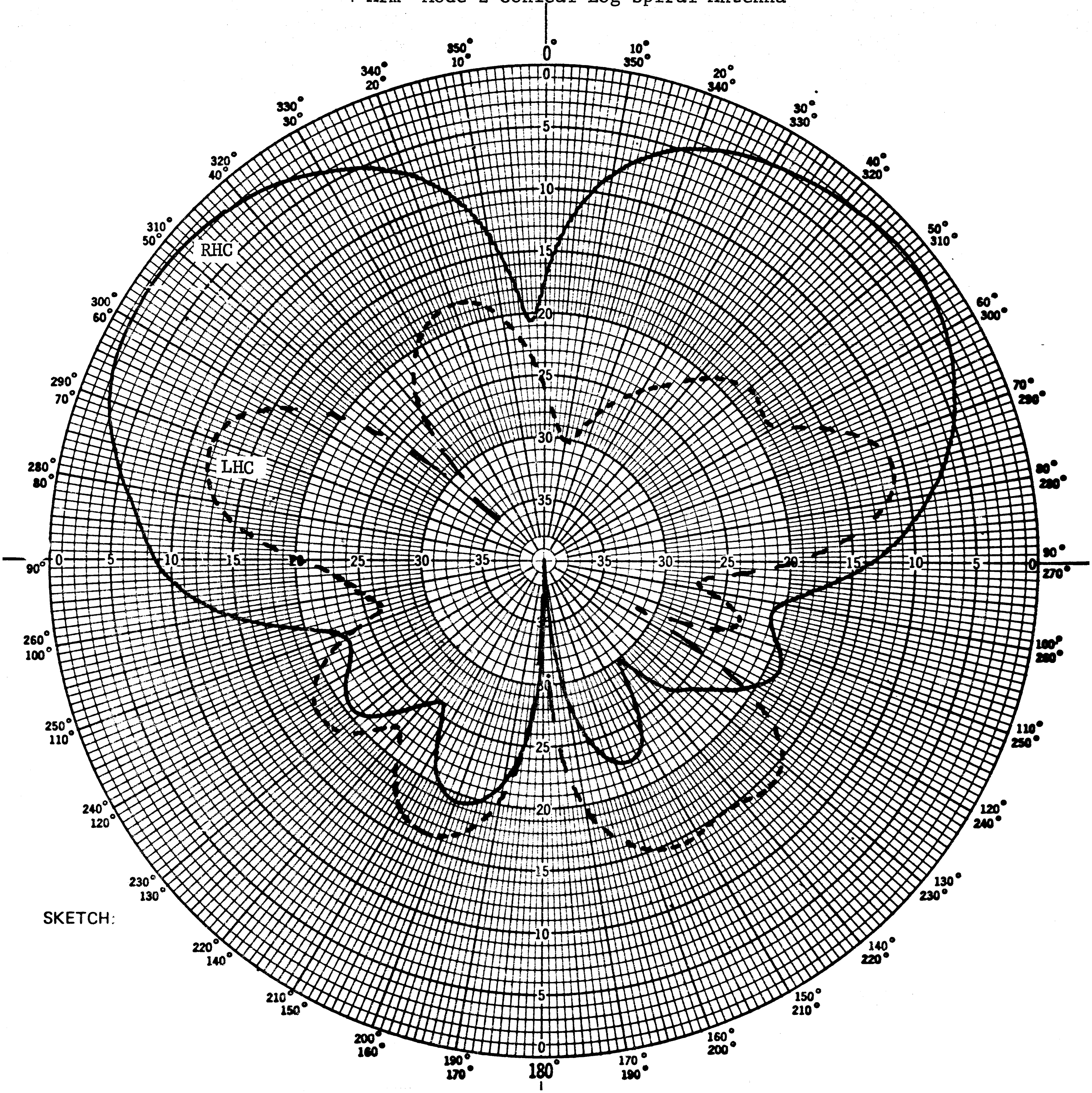
The four arm spiral can also be fed in the T_1 mode if the phase shift between windings is 90° as shown above for RHC.

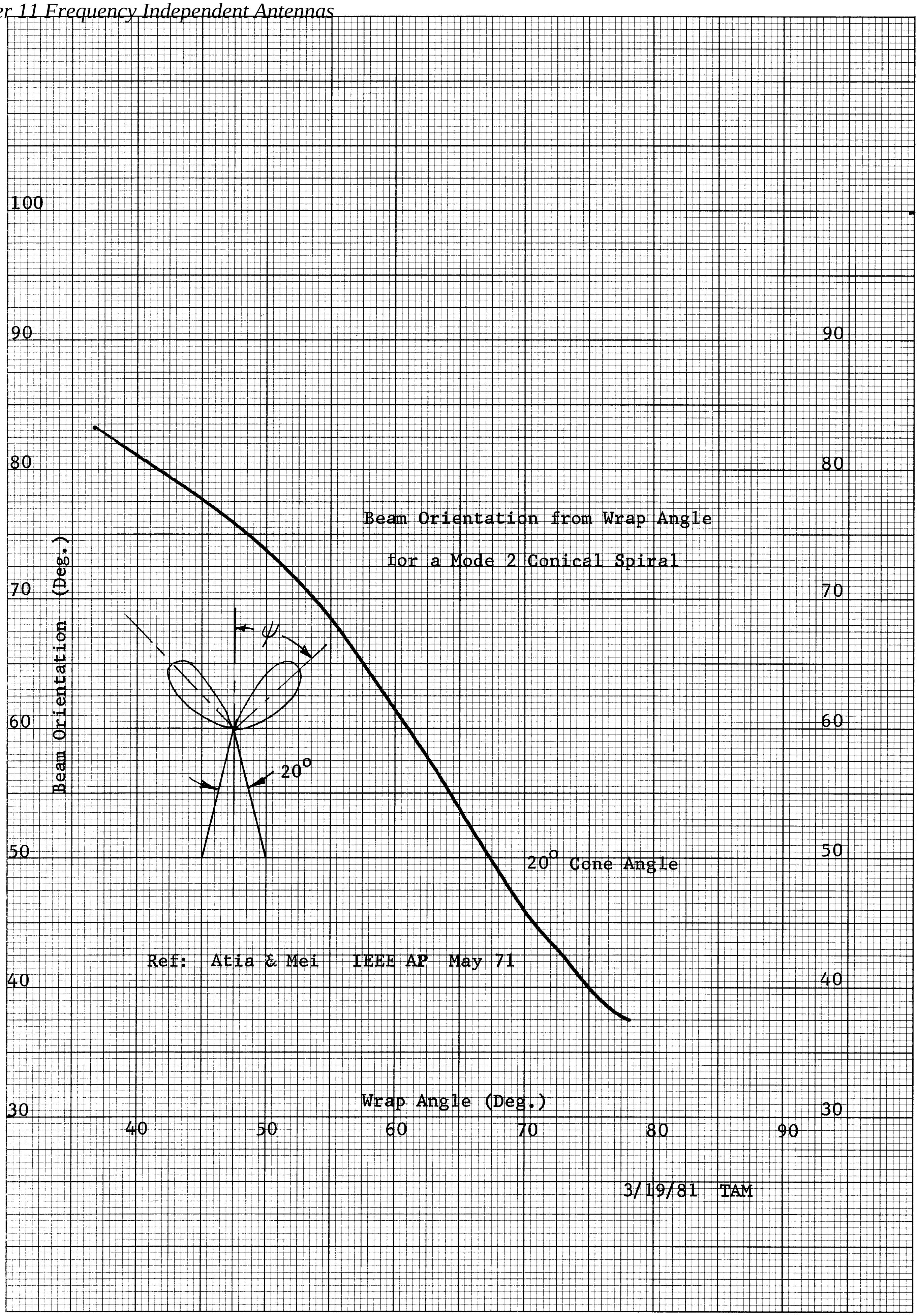
There is only limited design information in the literature. No one has done an extensive study like Dyson did for the two arm spiral. A typical measured pattern is given on page 454. The pattern has a null on boresight and broad beams pointing at about 52°. Both the limited theory and experience have shown that the beamwidth can be varied only over a very limited range of values. It will vary from about 48° to 60°. Higher wrap angles and smaller cone angles will decrease the beamwidth but only within the limits given above. The direction of the beam can be varied by changing the wrap angle of the spiral. On page 455 is a curve of beam direction versus the wrap angle. It has been found that this will hold true for a large variation of the cone angle. We have an antenna which we can vary the direction of the beam but not the beamwidth.

The diameters at the top and bottom of the cone will be larger than the two arm case. It has been found that if the lower truncation constant given by Dyson's curves for the two arm spiral are multiplied by 1.42 then this will be a good truncation constant for the four arm mode 2 conical spiral. This seems to hold for a range of wrap and cone angles. The upper trunca-

Martin Marietta Antenna Laboratory

4 Arm Mode 2 Conical Log Spiral Antenna




Beam Orientation from Wrap Angle
for a Mode 2 Conical Spiral
Beam Orientation (Deg.)
Wrap Angle (Deg.)
20° Cone Angle
Ref: Atia & Mei IEEE AP May 71
ψ
20°
100
90
80
70
60
50
40
30
40
50
60
70
80
90
3/19/81 TAM

tion constant should be multiplied by a factor which is a linear function of the wrap angle. It varies from 4 at 65^{o} wrap angle to 2.3 times at a wrap angle of 80^{o} from the values given on Dyson's curves. Since a little extra length at the top of the antenna will not degrade the pattern or increase the length significantly, it seems best to just use the lower value for all designs.

Example. Design a 4 arm mode 2 conical spiral to point the beam at 60^{o} from 500 MHz to 1500 MHz. Cone angle equal 10 degrees.

From the curve on page 455 we find that the wrap angle needs to be about 60^{o} for the pattern to point at 60^{o}. We find the upper and lower truncation constants from the curve on page 451 for the two arm spiral. To these values we multiply by the factors given above to obtain the truncation constants for the mode 2 spiral. The upper diameter is determined by the upper frequency and the lower diameter by the low frequency.

Upper Diameter = (2)(2.3)(.045)(11.80285/1.5) = 1.63 inches

Lower Diameter = (2)(1.42)(.25)(11.80285/0.5) = 16.76 inches

Height = 42.9 inches

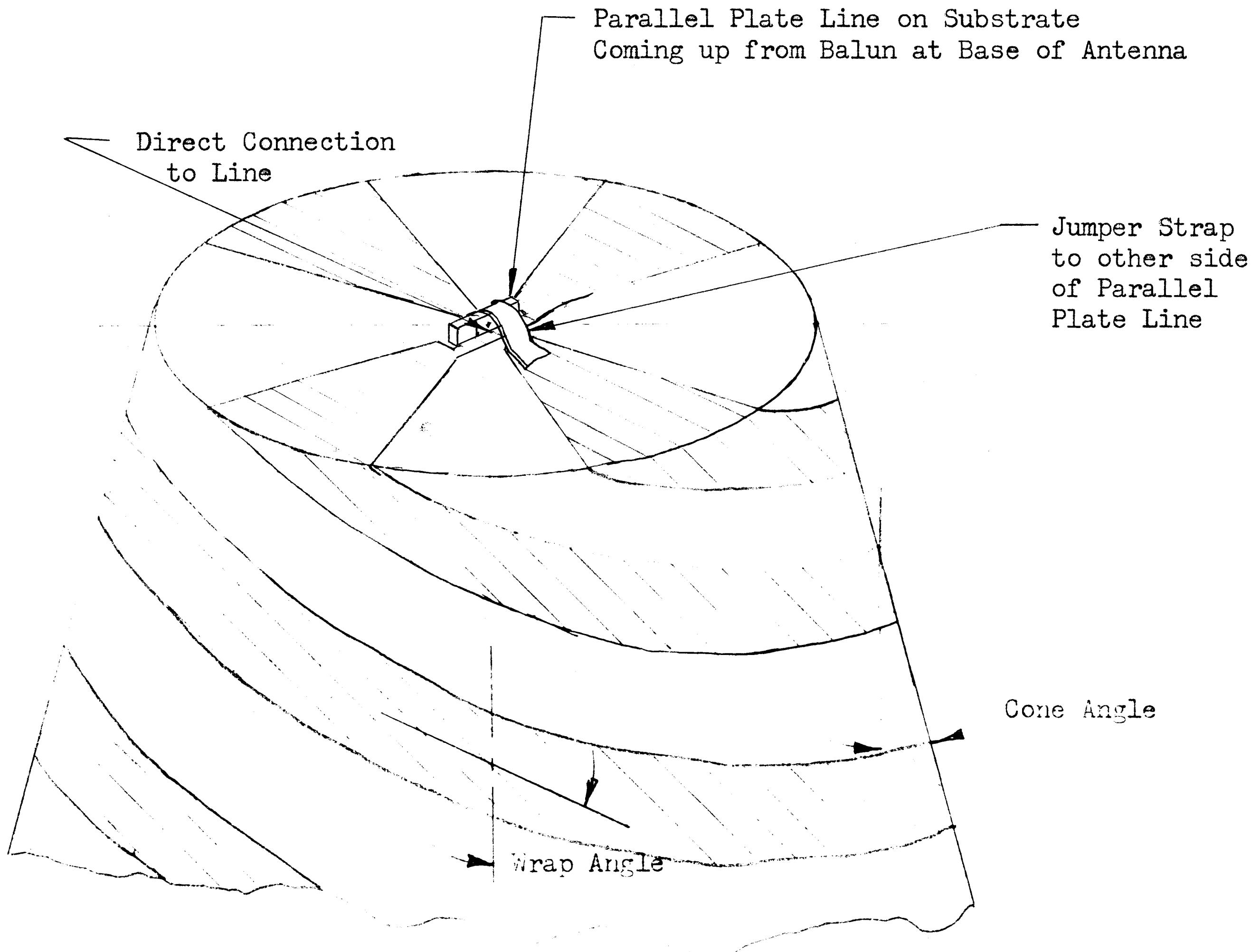
We could reduce the size of the antenna by using the curves for degraded performance at the bandedges.

FEEDING CONICAL LOG SPIRALS

The antennas must be fed with a balanced transmission line. The two arm spiral can be fed with the infinite balun in the same manner as the planar spiral. A coax line is soldered to one of the arms of the spiral. At the top the center conductor is connected to the other arm of the spiral which should have a dummy coax soldered to it to retain symmetry. The attenuation of the current beyond the active region will prevent currents induced on the outer shield of the coax from reaching the input. This balun requires a long section of coax line because the lengths of the arms are much longer than the planar spiral and the width of the windings must exceed the diameter of the coax. For some applications the loss in this long coax are objectionable.

The antenna may be also fed from a line upper the center line of the cone. A suitable balun for this application is the split tapered balun described on page 149. This balun can also be realized by tapering the ground plane of a microstrip line until the lines are balanced. A connection to this type of balun is drawn on page 457 for the 4 arm conical spiral. The two arm conical spiral can be directly connected to the two sides of the parallel plate line. A second type of balun which can be use on a line up the center of the cone is the ferrite bazooka balun described on page 147. With this type of balun, the coax can be run all the way to the connection which is shown on page 458 for the four arm conical spiral. It is a cleaner connection to the four arm spiral.

It is difficult to tell from the patterns if the four arm mode conical spiral needs a balun. The pattern does not squint in the same manner as the two

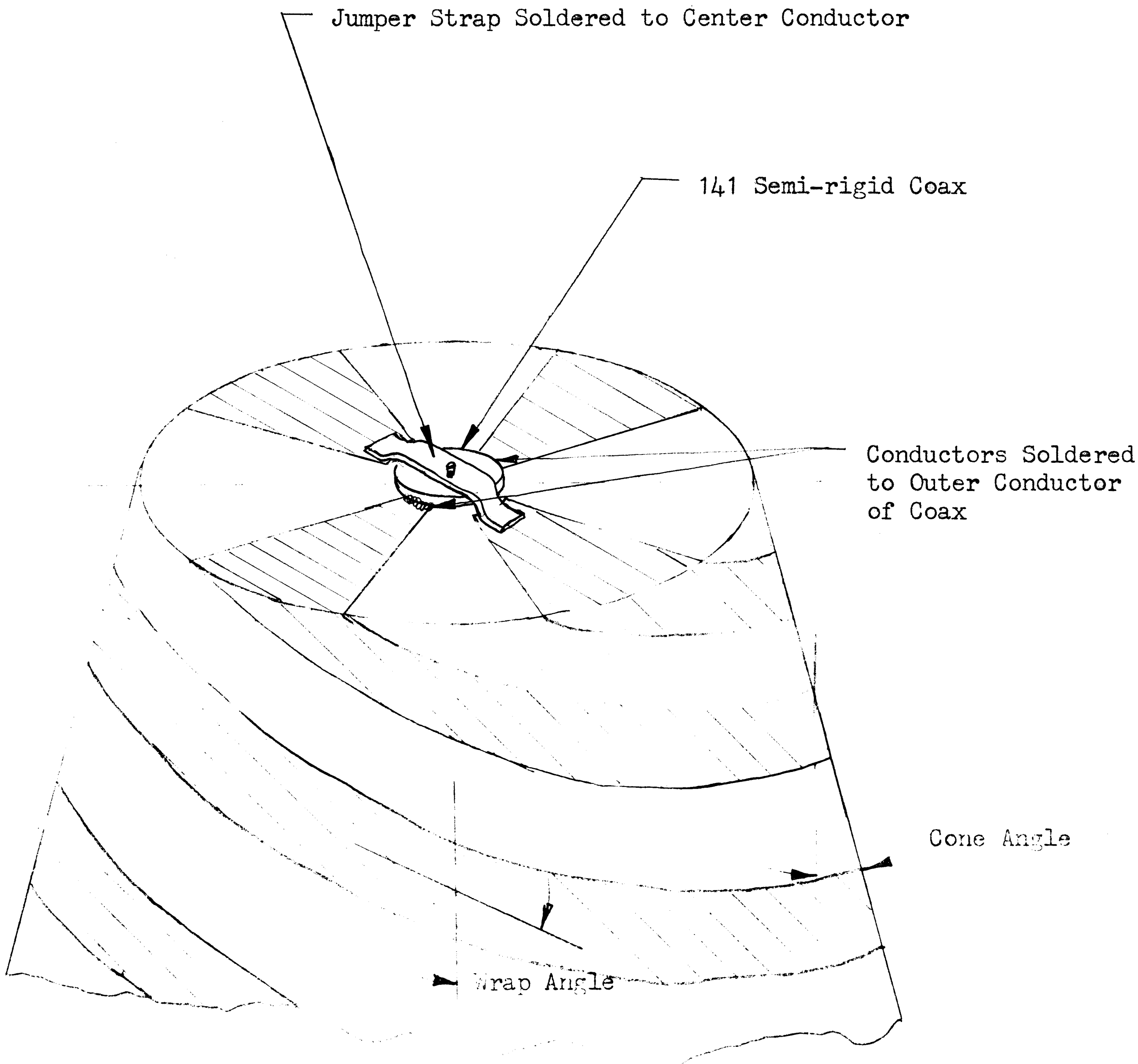


4 Arm Mode 2 Conical Log Spiral Connection to

Parallel Plate Line on Substrate running to

Tapered Microstrip Transmission Line Balun

Located at the base of the Antenna



4 Arm Mode 2 Conical Log Spiral Connection

to Coax feed without Balun

arm conical spiral. Since the pattern has a null on boresight and broad beamwidths, the squint cannot be detected in azimuth patterns. The squint shows in conical patterns about the beam peak. The pattern will show large ripple in these patterns when there should not be any. Like all squint problems, these pattern problems are caused by multi-modes on the antenna. In some applications the pattern of the 4 arm mode 2 spiral can have this ripple and still meet the requirements. The coax can be connected directly to the spiral without a balun.

Dyson has shown that the best patterns are obtained from a complementary antenna. If the arms are complementary with the spacings between them, then we can estimate the input impedance from the Babinet-Booker principle even though the antenna is not planar. The two arm spiral will be close to 189 ohms for the complementary antenna and the four arm impedance will be close to 95 ohms. Because the antenna is not planar, the impedances that are measured on the antennas tend to be somewhat lower. The two arm spiral usually measures about 150 ohms and the four arm conical spiral measures about 85 ohms. The width of the arms can be varied to obtain a better match with only a little effect on the pattern responses. Larger widths on the arms will reduce the input impedance.

LOG PERIODIC ANTENNAS

The number of structures which will continuously scale themselves is quite limited. All these structures involve the rotation of the observation point to achieve frequency independent operation. They also radiate circular polarization. Linearly polarized frequency independent antennas are achieved by using structures which scale exactly only at discrete frequencies and are not strictly frequency independent. Between the frequencies of the scaling, the pattern characteristics will ripple between the exact scaling frequencies. If the scalings are close in relative bandwidth, then the antenna may not change its pattern and impedance response rapidly and the antenna is practically frequency independent.

Not only do we need a structure which will scale itself at discrete frequencies, but the current must attenuate rapidly outside an active region so that the antenna can be truncated. Truncation is possible if the parts of the antenna couple to each so that there is a change of modes in the antenna or if the parts of the antenna outside the active region do not resonate and the currents are reduced. The structure must also transmit the energy from a feed region to the active region. Before the active region is reached, it is a transmission line.

Every log periodic structure has a basic cell which is scaled. The ratio of scaling is a constant throughout the antenna.

$$\frac{f_1}{f_2} = \frac{\lambda_2}{\lambda_1} = \tau$$

τ is the scaling constant which will be taken as less than one. The antenna will have the same characteristics at the scaled frequency for an infinite structure. The antenna will scale exactly for the following sequence of frequencies.

$$f_n = \frac{f_0}{\tau^n}$$

The scaling period is related to the frequencies by

$$n \operatorname{Ln} \tau = \operatorname{Ln}(f_0/f_n)$$

The antenna is periodic in the logarithm of frequency. In fact every dimension of the antenna is log periodic since the dimensions are proportional to wavelength which is log periodic.

The design of log periodic antennas proceeds in two parts. The pattern characteristics are determined by the number of elements in the active region and their spacings. The (H Plane) pattern is also effected by the spacing of the different portions of the active region. The actual size of the antenna is determined by truncation constants and the frequency limits of operation. The truncation constants are determined from the number of elements in the active region. The possible range of gains achieveable is limited because there is a limit to how few elements are in the active region and still allow truncation and because the antenna is frequency independent which means the possible aperture size is limited.

We will cover the log periodic dipole antenna (LPDA) first and will use it to understand the other types of log periodic antennas.

LOG PERIODIC DIPOLE ANTENNA (LPDA)

The drawing on page 462 shows the elements of the log periodic dipole antenna. There are three dimensions: L_n is the length of the dipole elements, d_n is the spacing between elements, and R_n is the distance from the virtual apex to the dipole element. As mentioned above, all parts of the antenna have the same scaling constant, τ.

$$L_n = L_1 \tau^{n-1}$$

$$d_n = d_1 \tau^{n-1}$$

$$R_n = R_1 \tau^{n-1}$$

Note that d_n is not an independent variable.

$$d_n = R_n - R_{n+1} = R_n(1-\tau)$$

Since the ratio of L_n to R_n is the same for all elements,

$$\frac{L_n}{R_n} = \frac{L_1 \tau^{n-1}}{R_1 \tau^{n-1}} = \frac{L_1}{R_1}$$

the end points of the elements will lie on the same line as shown. The angle between the endpoints and the center line is called the half apex angle, α. This angle is given by

$$\alpha = \tan^{-1}\left(\frac{L_n}{2R_n}\right)$$

Carrel[1] introduced a second constant of the log periodic dipole antenna to describe the spacing between elements.

$$\sigma = \frac{d_n}{2L_n} \qquad \text{Spacing Constant}$$

The LPDA will then be determined by the two constants:

τ - Scaling Constant $\qquad$ σ - Spacing Constant

All antennas with the same constants will have the same patterns (same Phi angles). The half apex angle can be determined from τ and σ.

$$L_n = \frac{d_n}{2\sigma} = \frac{R_n(1-\tau)}{2\sigma}$$

Carrel, R., "An Analysis of the Log Periodic Dipole antenna", 10th Annual Symposium on USAF Antenna R & D Program, October, 1960.

$$\alpha = \tan^{-1}\left(\frac{R_n(1-\tau)}{4R_n\sigma}\right) = \tan^{-1}\left(\frac{1-\tau}{4\sigma}\right)$$

This function has been reduced to a nomograph on page 464. The values on the scales go beyond the practical limits of values for good designs. Of course, the chart should help eliminate designs with small half apex angles which gives very long antennas.

Strictly speaking the diameters of the dipoles should also be scaled by the scaling constant. The antenna may be successfully made without scaling the diameters or by scaling in steps. Notice that the feed lines alternate between dipole elements, the extra 180° phase shift per element is needed for the truncation requirements. We will come back to the reasons for this and now proceed to the design.

Similiar to the work of Dyson on the spiral antenna, Carrel measured and calculated a large number of antennas. On pages 465 and 466 are plots of the E and H plane beamwidths of the LPDA for various parameters. In general the E plane beamwidth is less than the H plane. Because the antenna is made from dipoles, there will be a null at $\theta = 90^{\circ}$ in the E plane. Using these beamwidths, Carrel[1] calculated the directivity using Kraus' formula, page 35, and produced a curve given on page 467. Suitable values of the parameters can be found from this curve and the estimated directivity can be determined for given designs. Since there is a peaking of gain on the curve, there is a line of optimum designs: minimum scaling constant for a given directivity. This is the first part of the design; the size is determined by truncation constants.

The truncation constants can be found from near field measurements or from calculated values of the currents in the elements similiar to the conical spiral. The lower truncation constant is determined from the low frequency end of the active region and the upper from the high frequency end currents. The antenna can be truncated beyond these points without effecting the pattern response. Like all truncation constants, the exact point will be vague. They are determined by how much pattern distortion is acceptable. Two curves of the estimated truncation constants have been drawn on page 468. The curve for K_1, the lower truncation constant, was drawn for the worst case spacing constant, σ. The lower truncation constant is about 0.5 . This constant is the required length of the longest element, L_1, at the lowest operating frequency.

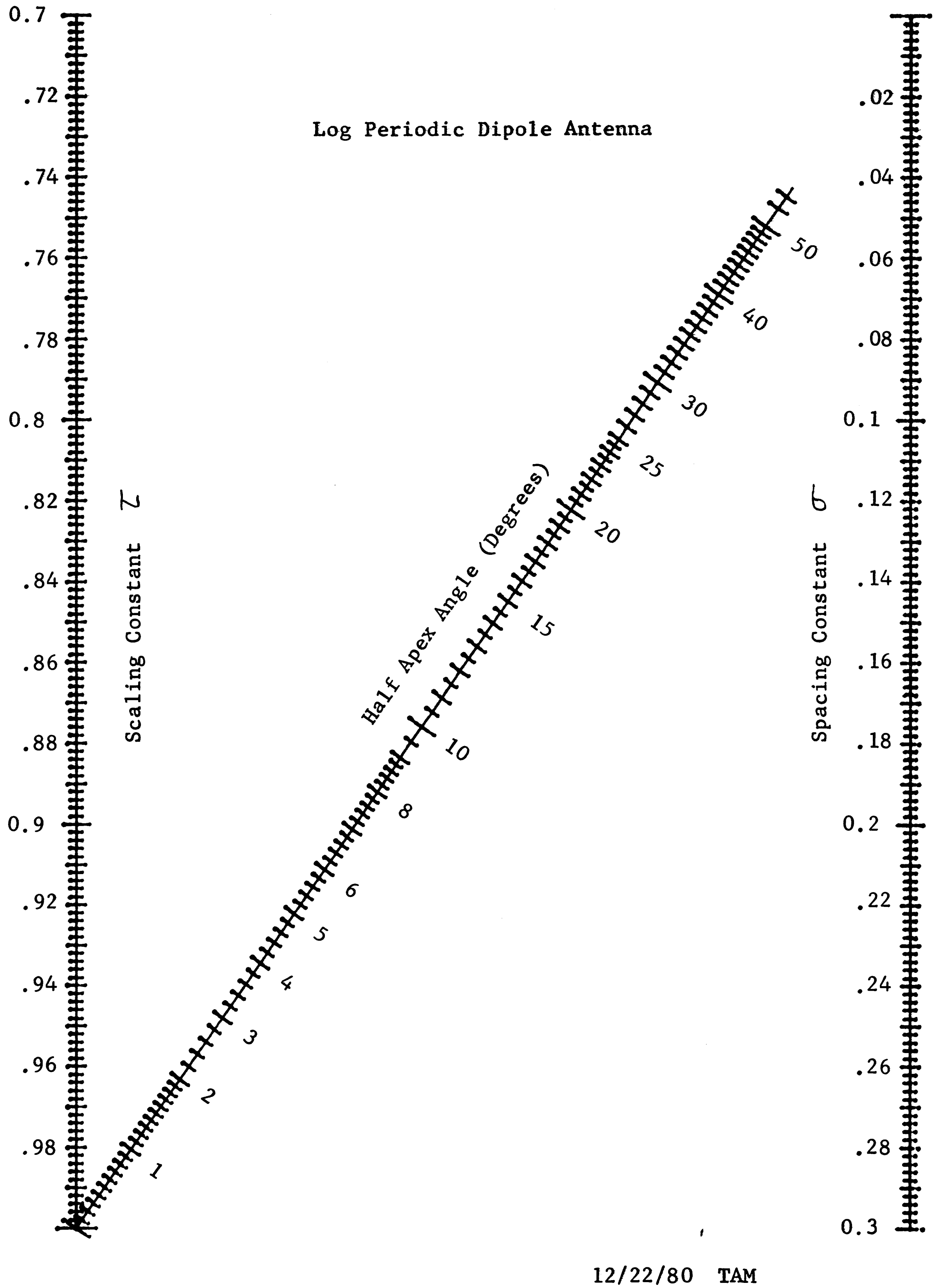
$$L_1 = K_1 \lambda_L$$

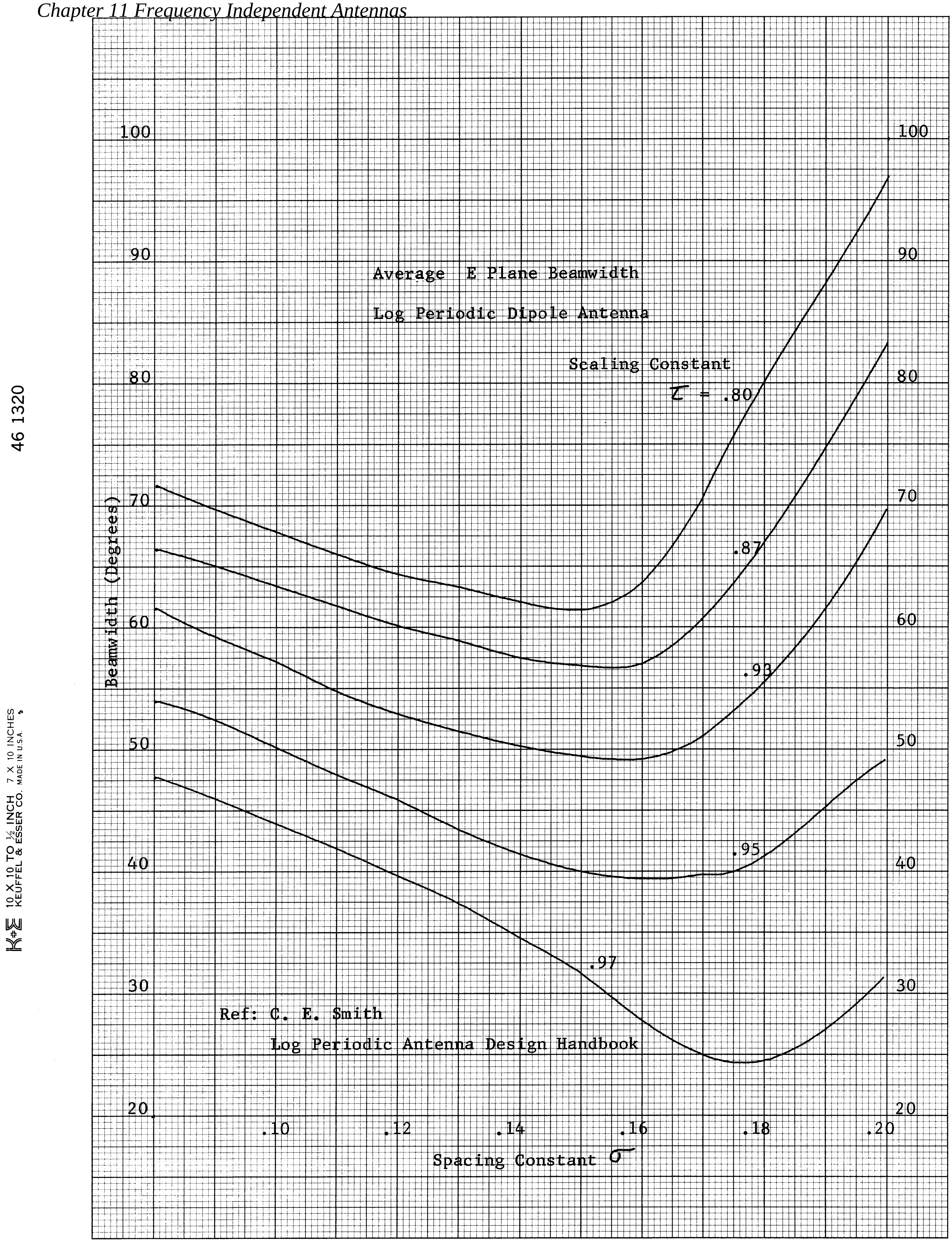
Where λ_L is the wavelength of the lowest frequency. The smallest element is determined by the upper truncation constant, K_2, and the upper operating frequency bandedge.

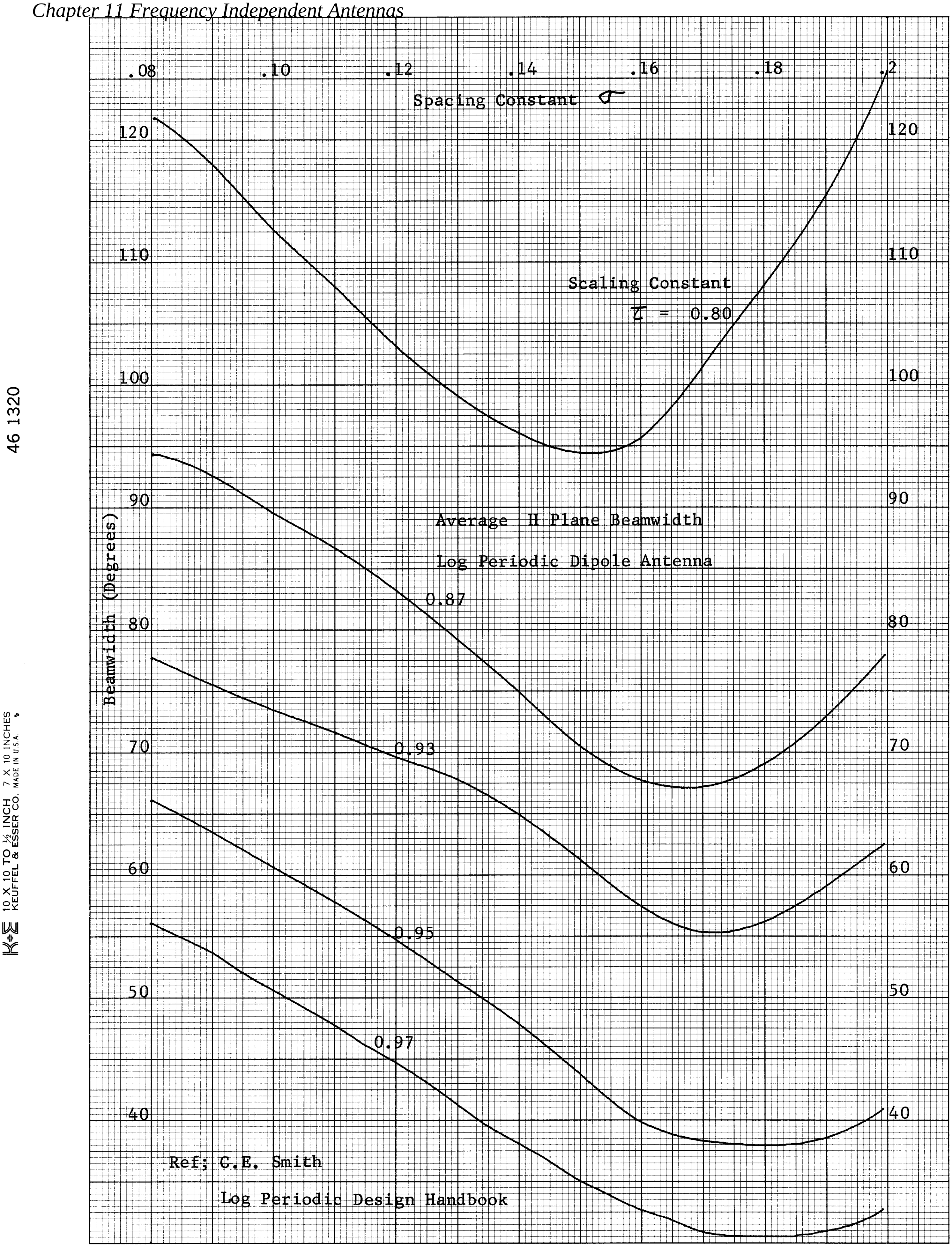
$$L_u = K_2 \lambda_U$$

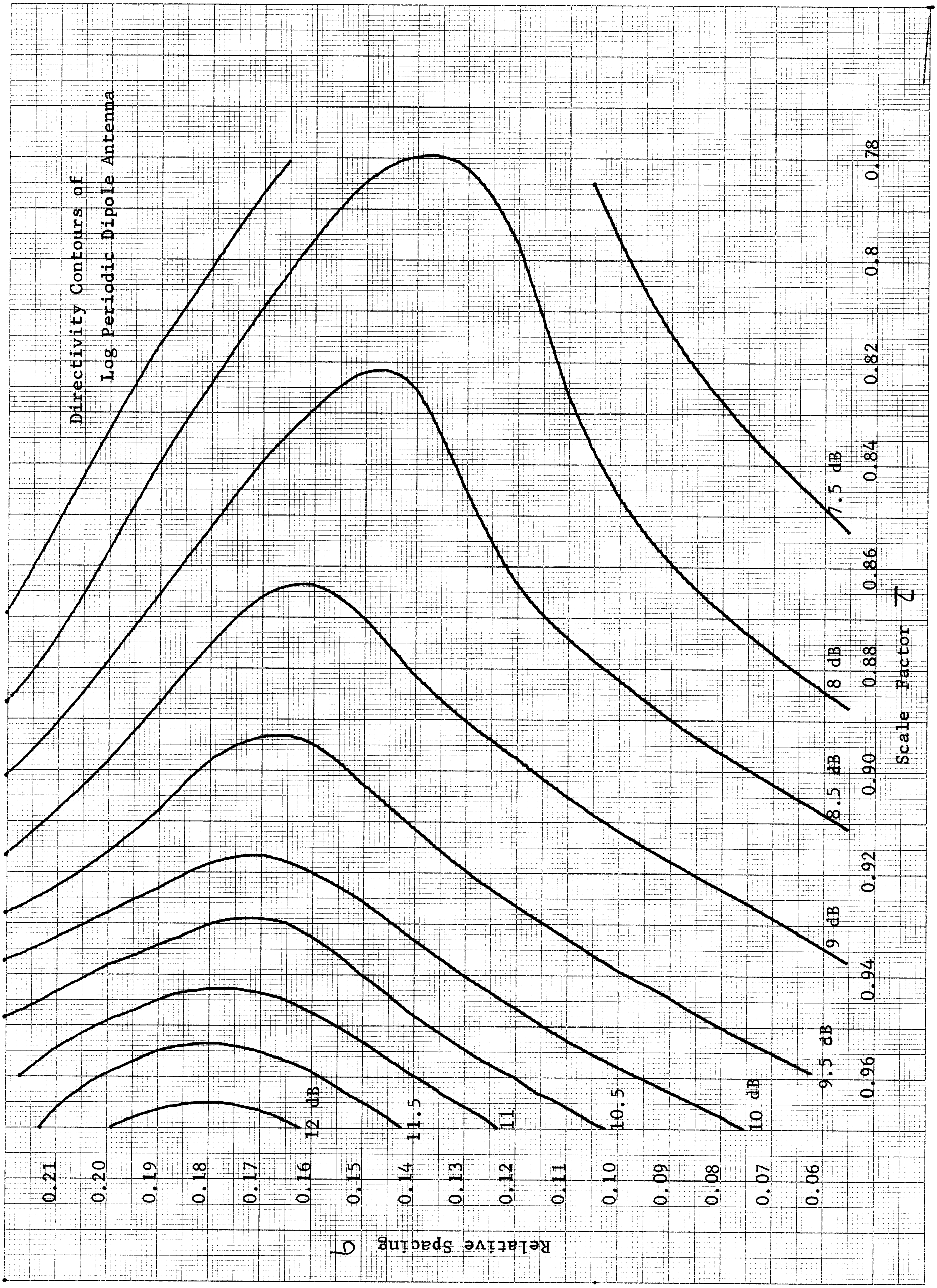
Where λ_U is the wavelength of the highest frequency. The largest and smallest elements are related by the scaling constant, τ, and the number of

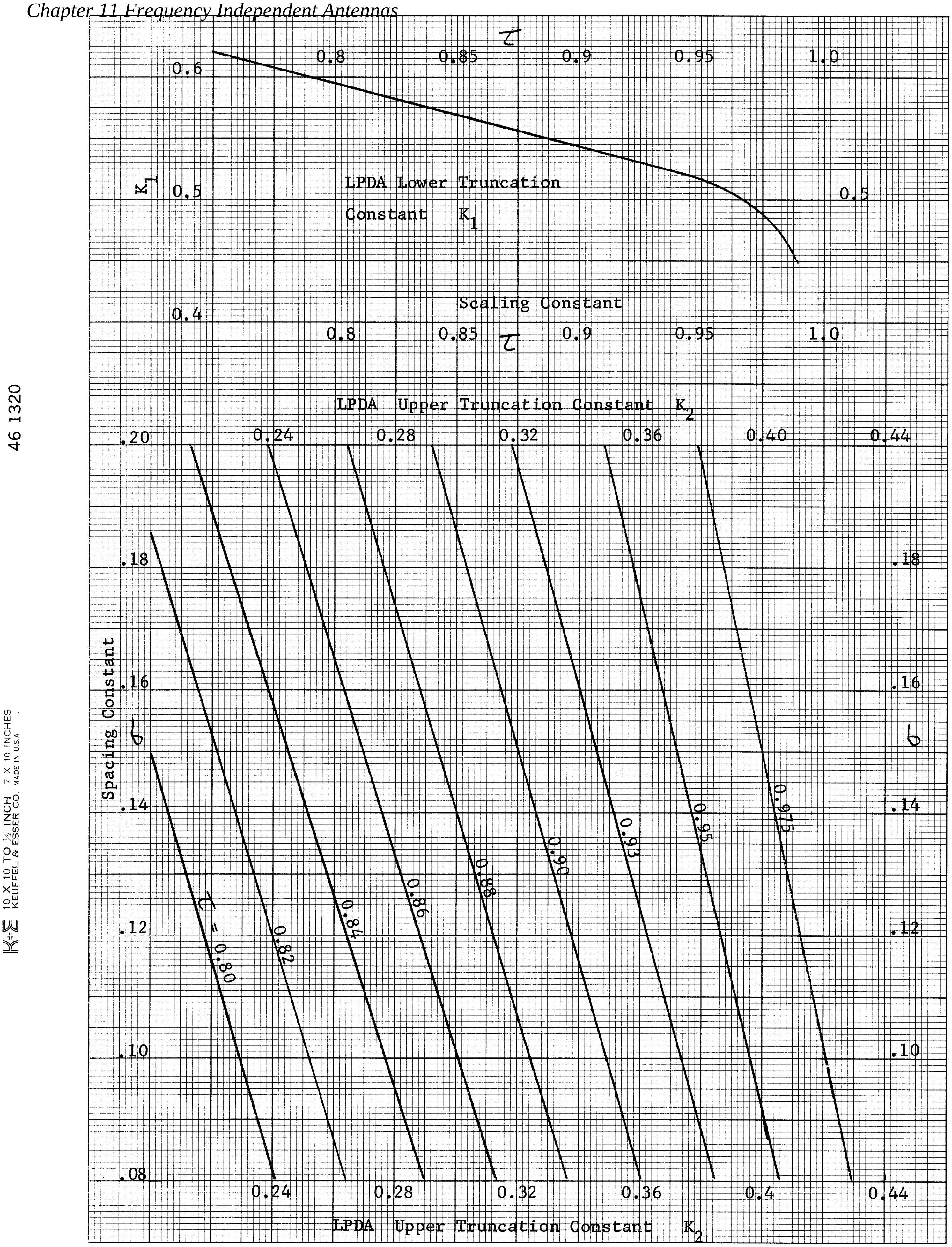
Carrel, R.L., "The Design of Log-Periodic Dipole Antennas", 1961 IRE International Convention Record.


Log Periodic Dipole Antenna
0.7
.72
.74
.76
.78
0.8
.82
.84
.86
.88
0.9
.92
.94
.96
.98
Scaling Constant τ
Half Apex Angle (Degrees)
1
2
3
4
5
6
8
10
15
20
25
30
40
50
Spacing Constant σ
.02
.04
.06
.08
0.1
.12
.14
.16
.18
0.2
.22
.24
.26
.28
0.3
12/22/80 TAM


Average E Plane Beamwidth
Log Periodic Dipole Antenna
Scaling Constant
τ = .80
.87
.93
.95
.97
Ref: C. E. Smith
Log Periodic Antenna Design Handbook
Beamwidth (Degrees)
Spacing Constant σ
100
90
80
70
60
50
40
30
20
.10
.12
.14
.16
.18
.20
46 1320
10 X 10 TO ½ INCH 7 X 10 INCHES
KEUFFEL & ESSER CO. MADE IN U.S.A.


.08
.10
.12
.14
.16
.18
.2
Spacing Constant σ
120
110
100
90
80
70
60
50
40
Scaling Constant
τ = 0.80
Average H Plane Beamwidth
Log Periodic Dipole Antenna
0.87
0.93
0.95
0.97
Beamwidth (Degrees)
Ref; C.E. Smith
Log Periodic Design Handbook
46 1320
10 X 10 TO ½ INCH 7 X 10 INCHES
KEUFFEL & ESSER CO. MADE IN U.S.A.


Directivity Contours of
Log Periodic Dipole Antenna
7.5 dB
8 dB
8.5 dB
9 dB
9.5 dB
10 dB
10.5
11
11.5
12 dB
0.78
0.8
0.82
0.84
0.86
0.88
0.90
0.92
0.94
0.96
Scale Factor τ
0.21
0.20
0.19
0.18
0.17
0.16
0.15
0.14
0.13
0.12
0.11
0.10
0.09
0.08
0.07
0.06
Relative Spacing σ


τ
0.6
0.8
0.85
0.9
0.95
1.0
K1
0.5
LPDA Lower Truncation
Constant K1
Scaling Constant
0.4
LPDA Upper Truncation Constant K2
.20
0.24
0.28
0.32
0.36
0.40
0.44
.18
.16
.14
.12
.10
.08
Spacing Constant
σ
τ = 0.80
0.82
0.84
0.86
0.88
0.90
0.93
0.95
0.975
0.4
46 1320
10 X 10 TO ½ INCH 7 X 10 INCHES
KEUFFEL & ESSER CO. MADE IN U.S.A.

elements, N. Using these truncation constants we can find the required number of elements.

$$L_u = L_1 \tau^{N-1} = L_1 10^{(N-1)\log\tau}$$

$$N = 1 + \frac{\log(L_u/L_1)}{\log\tau}$$

This can be related to the truncation constants, and the upper and lower frequency bandedges.

$$\frac{L_u}{L_1} = \frac{K_2\lambda_u}{K_1\lambda_L} = \frac{K_2 F_L}{K_1 F_u}$$

$$N = 1 + \frac{\log\left(\frac{K_2}{K_1}\frac{F_L}{F_u}\right)}{\log\tau}$$

Where F_U and F_L are the upper and lower frequencies. The number of elements is a function of the ratio of the truncation constants, the relative bandwidth of the antenna, and the scaling constant, .

The truncation constants define the edges of the active region. If we pick a single frequency, then the constants can be used to find the number of elements in the active region.

$$N_a = 1 + \frac{\text{Log}(K_2/K_1)}{\text{Log}(\tau)}$$

In general more elements in the active region means higher gain and less ripple in the frequency response. But this is true because the scaling constant, τ, is higher. These relationships have been reduced to a nomogram on page 470. Using this graph, the number of elements in the LPDA can be determined.

The spacings between elements are found by using the element length, L_n, and the spacing constant, σ.

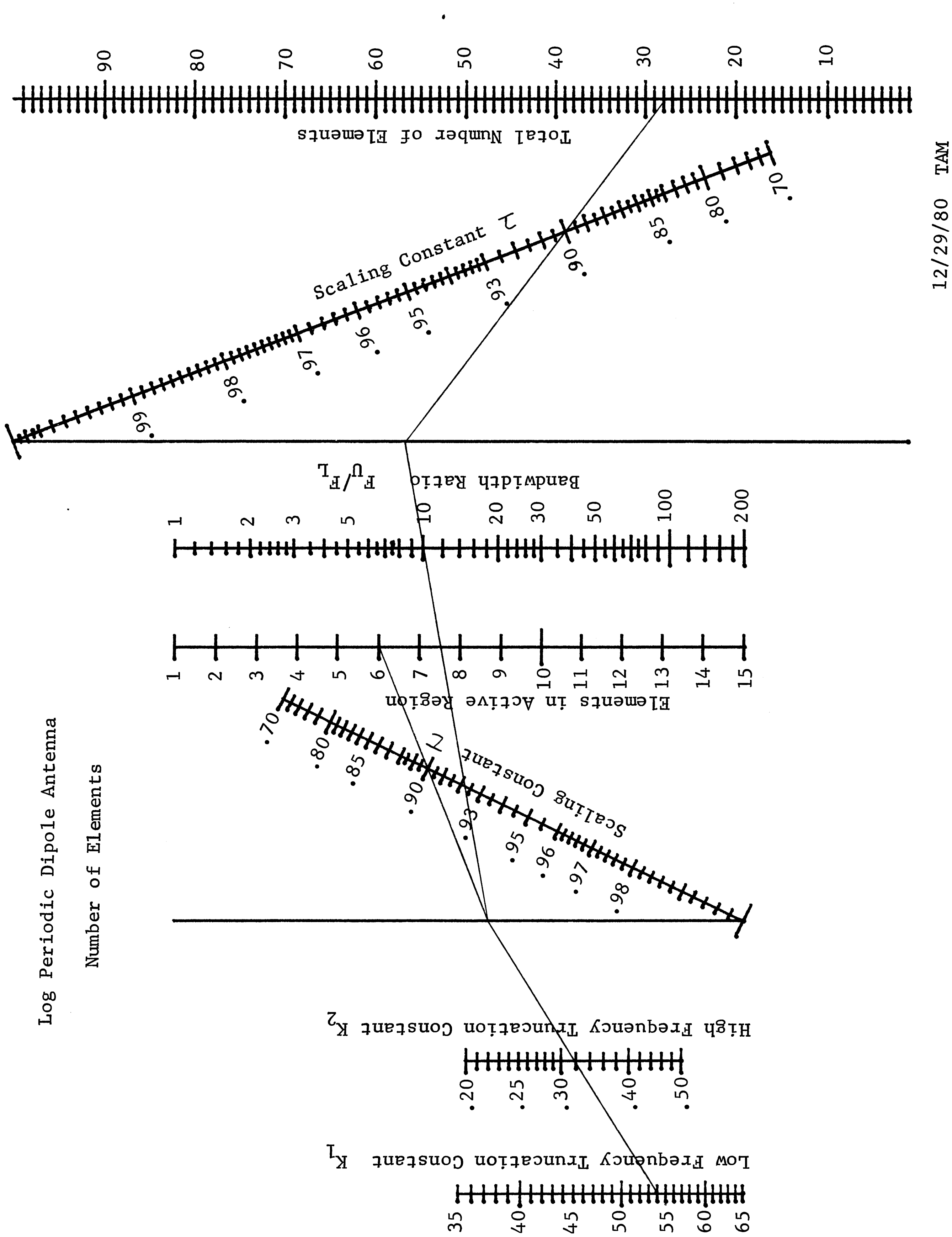
$$d_n = 2 L_n \sigma$$

The distance from the element to the virtual apex is given by

$$R_n = \frac{d_n}{1-\tau} = \frac{2 L_n \sigma}{1-\tau} = \frac{L_n}{2\tan\alpha}$$

α is the half apex angle. The axial length of the total antenna can be found by using these R_n.

$$\text{Length} = R_1 - R_N = R_1(1-\tau^{N-1}) = \frac{2 L_1 \sigma (1-\tau^{N-1})}{1-\tau}$$


Log Periodic Dipole Antenna
Number of Elements
Total Number of Elements
Scaling Constant τ
Bandwidth Ratio F_U/F_L
Elements in Active Region
Scaling Constant τ
High Frequency Truncation Constant K_2
Low Frequency Truncation Constant K_1
12/29/80 TAM

470

Example. Design a LPDA from 100 MHz to 1000 MHz with $\tau = 0.9$ and $\sigma = .15$

The E and H plane beamwidths can be estimated from the curves on pages 465 and 466.

$$\text{E Plane Beamwidth} = 53^o$$

$$\text{H Plane Beamwidth} = 66^o$$

The directivity is given on page 467 as about 9.5 dB. The size of the antenna is determined from the frequency bandedges and the truncation constants. These are found on page 468.

$$\text{Lower Truncation Constant, } K_1 = 0.54$$

$$\text{Upper Truncation Constant, } K_2 = 0.32$$

The longest element, L_1, is found from the lower truncation constant and the low frequency bandedge, 100 MHz.

$$L_1 = K_1 \lambda_{100\text{ MHz}} = 0.54 \frac{11.80285}{0.1} = 63.74 \text{ inches}$$

We can find the number of elements in the active region and the required number of elements from the nomogram on page 470. Draw a line intersecting the two values of the truncation constants to the blank axis on the left side of the paper. From this point on the blank axis draw a line through the value of the scaling constant on the tilted axis to the right to the next vertical axis which is the number of elements in the active region. In this example there are 6 elements in the active region. Next draw a line from the point on the blank axis on the left through the bandwidth ratio scale to the blank axis on the right. The bandwidth ratio in the example is 10. Draw a line from the point on the blank axis to the right through the value of the scaling constant on the tilted axis to the last scale to the right. On this scale read the total number of elements required in the antenna. The number required for the antenna in the example is 28.

The half apex angle is found from the formula on page 463.

$$\alpha = \tan^{-1}((1 - \tau)/4\sigma) = 9.46^o$$

The angle can also be found from the nomogram on page 464.

The distance of the first element from the virtual apex is found from the formula on the bottom of page 469.

$$R_1 = \frac{2 L_1 \sigma}{1 - \tau} = 191.21 \text{ inches}$$

Notice that the first element is the longest element even though the antenna will be fed from the end with the shortest element. Using this value of R_1, we can find the total length of the antenna.

$$\text{Length} = R_1(1 - \tau^{N-1}) = 180 \text{ inches}$$

The initial spacing between the element is found from the scaling constant, τ, and the length of the first element, L_1.

$$d_1 = 2 L_1 \sigma = 19.12 \text{ inches}$$

The lengths on the elements, the spacings between them, and the distances from the virtual apex can be found by successive multiplications by the scaling constant. When this is done the following table is generated with the design.

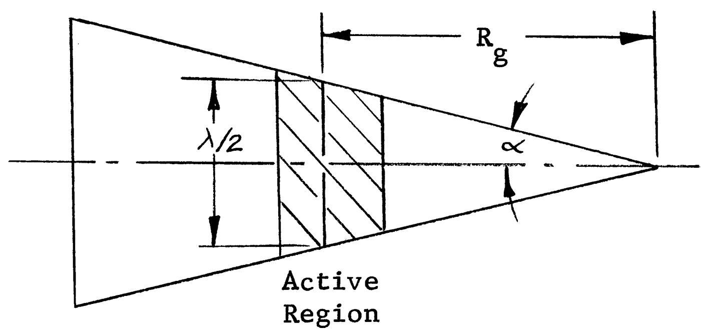
$$L_n = L_{n-1}\tau \qquad d_n = d_{n-1}\tau \qquad R_n = R_{n-1}\tau$$

Number	Element Length	Spacing	Virtual Apex Distance
1	63.74	19.12	191.21
2	57.37	17.21	172.09
3	51.63	15.49	154.88
4	46.47	13.94	139.39
5	41.82	12.54	125.45
6	37.64	11.29	112.91
7	33.87	10.16	101.62
8	30.49	9.15	91.46
9	27.44	8.23	82.31
10	24.69	7.41	74.08
11	22.22	6.67	66.67
12	20.00	6.00	60.00
13	18.00	5.40	54.00
14	16.20	4.86	48.60
15	14.58	4.37	43.74
16	13.12	3.94	39.37
17	11.81	3.54	35.43
18	10.63	3.19	31.89
19	9.57	2.87	28.70
20	8.61	2.58	25.83
21	7.75	2.32	23.25
22	6.97	2.09	20.92
23	6.28	1.88	18.83
24	5.65	1.69	16.95
25	5.08	1.53	15.25
26	4.58	1.37	13.73
27	4.12	1.24	12.35
28	3.71		11.12

PHASE CENTER

We first discussed phase center when we covered corrugated horns on page 284. It is that point on the antenna where it appears that the antenna radiates spherical waves. This phase center can only be defined for the main beam. Even then the phase center will only be an average. The method of measuring has been discussed before. For most antennas the location will depend on the plane of measurement (E or H Plane).

The phase center of the LPDA is approximately the same in both planes. We are interested in the phase center of the antenna because it is very useful as a feed for a parabolic reflector. The phase center is located approximately at that point on the antenna where there would be an element a half wavelength long. We will measure the distance from the virtual apex.



The distance to the half wave element is $R_g = \frac{\lambda}{4 \tan\alpha}$

Carrel found the phase center analytically and related it to R_g

$$K = \frac{R_p}{R_g}$$

This ratio is plotted on page 474. Notice that the E and H plane factors are different and that the phase center location is only a function of the scaling constant, τ. The phase center is slightly in front of the half wave element location.

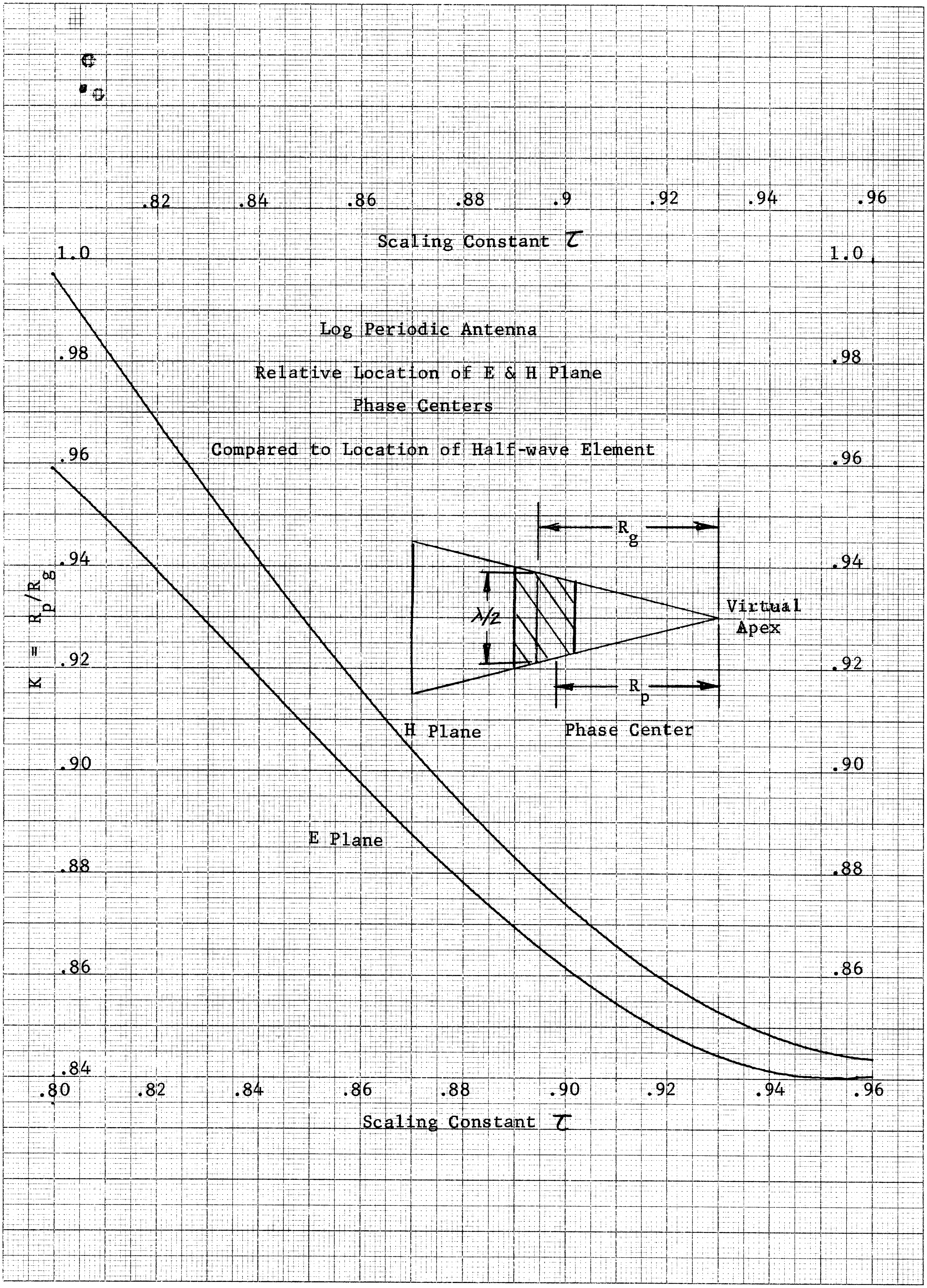
$$R_p = \frac{K\lambda}{4 \tan\alpha}$$

Let us calculate the phase center of the example above at the geometric mean frequency of 316 MHz. From the plot we find

$$K_e = .8622 \qquad K_h = .874$$

$$R_e = \frac{11.80285\ K_e}{(.316)4\ \tan(9.46)} = 47.86 \text{ inches} \qquad R_h = 48.52$$

We must be satisfied with a compromise when the antenna is used as a feed for a reflector. The phase center of the feed should be located at the focus of the parabola, but the phase center moves with frequency. On solution is to use small values of σ, the spacing constant, which corresponds to large


.82 .84 .86 .88 .9 .92 .94 .96
Scaling Constant τ
1.0
Log Periodic Antenna
Relative Location of E & H Plane
Phase Centers
Compared to Location of Half-wave Element
.98
.96
.94
.92
.90
.88
.86
.84
K = R_p/R_g
R_g
λ/2
Virtual Apex
R_p
H Plane
Phase Center
E Plane
.80 .82 .84 .86 .88 .90 .92 .94 .96
Scaling Constant τ

values of α, the half apex angle. Then the phase center moves slower with changes in frequency and the phase error does not increase due to axial defocusing as fast as feeds with small α.

$$\frac{dR_p}{df} = \frac{-ck}{4f^2 \tan\alpha} \qquad \frac{dR_p}{d\lambda} = \frac{K}{4\tan\alpha}$$

The rate of change of the phase center with respect to wavelength is a constant independent of wavelength (the antenna scales itself).

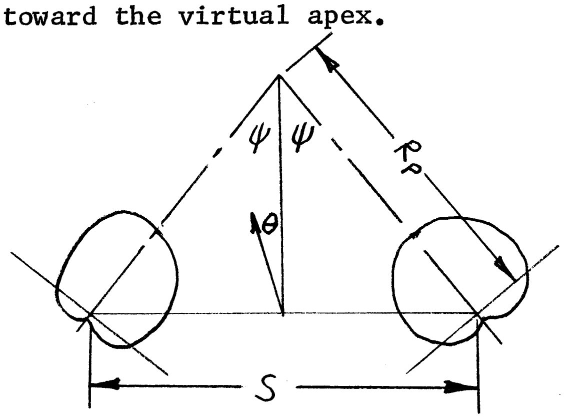
ELEVATION ANGLE (Psi)

The dipole elements of the antenna must be alternately connected with a phase reversal between the elements. The usual way of achieving this is to run two lines for a feed and alternately connect the dipoles to it as shown in the figure on the top of page 476. It is not necessary that the two sides remain separated by the same amount. We can space them apart at an angle as shown on the bottom of page 476. For the antenna to remain frequency independent it is necessary for the two sides to intersect at the virtual apex. The angle is measured from the center line between the sides and is denoted: ψ. The phase center is a linear function of wavelength when measured from the virtual apex. If the two sides are angled from the virtual apex, then the phase center distance from the apex will scale exactly with frequency and the distance from the two portions of the active region on the two sides remains constant in wavelengths.

The main effect of the angle ψ is to decrease the H plane beamwidth as increases. This can be understood by considering the sides as elements of an array. As we increase the distance between the array elements, the beamwidth decreases. The distance is twice the distance from the phase center location to the center line.

$$S = 2\,R_p \sin\psi$$

We can analyze the pattern response if we assume there are two elements with patterns aligned toward the virtual apex.



$$E(\theta) = e^{j\frac{\beta S}{2}\sin\theta}\cos^n((\theta+\psi)/2) + e^{-j\frac{\beta S}{2}\sin\theta}\cos^n((\theta-\psi)/2)$$

Above is an approximate expression for the pattern in the H plane. The value of n is determined from the beamwidth of the antenna when $\psi = 0$. From page 36 we have

$$n = \frac{\text{Log}(\frac{1}{2})}{2\ \text{Log}(\text{Cos}(\text{HPBW}/4))} \quad \text{HPBW - Half Power Beamwidth}$$

This is an array where the elements do not have the same pattern so pattern multiplication cannot be used. The patterns are the same but point in different directions. One trouble with this analysis is that the effects of mutual coupling have not been accounted for in the development. Because each side is only half an antenna, the coupling between sides is an important factor.

One consequence of moving the sides apart is to increase the backlobe and reduce the F/B. In the limit as $\psi = 90^{o}$, the front and backlobes are equal which we can see from symmetry. The distance of the phase center from the virtual apex will be reduced. The phase center will be on the center line between the sides. The phase center distance is now

$$R_{p\psi} = R_{po}\ \text{Cos}\,\psi$$

Where R_{po} is the phase center distance along one of the arms. This reduces the variation with frequency as well by the same factor.

Some designs use a variable ψ angle, increasing for lower frequencies. If we were to estimate the beamwidth using the formula above, we must use the distance between the phase center sections and not the expression involving sin ψ which assumes a unform ψ throughout the antenna. In these cases the phase center distance from the apex should be measured along the arm and not directly. The local rate of change of phase center is proportional to the local cos ψ.

ARRAYS OF LPDA

The antennas can be used to make broadband arrays. If we want to retain the frequency independent nature of the antenna element, then we must keep the distance between the phase centers the same in terms of wavelengths as the frequency changes. The array will be frequency independent only if the virtual apex of all the elements of the array coincide. Also the scaling constant, τ, and the spacing constant, σ, must be the same in all antennas of the array. On page 478 are diagrams of E and H plane arrays of LPDA. The phase center distance from the apex is a linear function of wavelength. In terms of wavelengths the distance between the phase centers of the elements of the array are equal for all frequencies. The pattern of the array must be found by a formula similiar to the one above where the direction of the pattern elements must be accounted for each element. The mutual coupling effects are less important because the second half of the antenna is near and the combination will cancel out the effects since they are out of phase.

The phasing of the antenna elements is important. If the antenna is turned over in the array, 180^{o} of phase will be added to the element. In an array of two elements this would give a null on the axis between them. This is the effect of placing an antenna over a ground plane. The virtual apex of an LP over a ground plane must be located on the ground plane. The log periodic antenna is unique because it can be phased in a frequency independent manner. Given a particular antenna, if we multiply every element by the scaling constant, then the phase at the phase center of the antenna will change by 180^o. This is the same antenna only the connections to the feeder have been changed. The location of the phase center does not change because there are more or less elements in front of it. It is located as a ratio of the point where an element would be a half wavelength. The phase can change arbitrarily by multiplying all the elements by

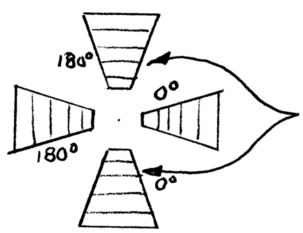
$$\tau^{\gamma/180} \qquad \gamma\text{- Phase Shift (Degrees)}$$

γ can be positive or negative. The number of elements does not directly determine the frequency bandedges but the truncation constants. This phase shift is relative to an antenna which remains the same. We can use this method only in an array. Changing the far field phase of a single antenna is irrelevant. To change the phase of one antenna relative to another by 90^o multiply by

$$\tau^{\frac{1}{2}}$$

If we have two antennas at right angles with one of them scaled by $\tau^{\frac{1}{2}}$, then we will have circular polarization when fed in phase. The opposite sense of circular polarization is obtained by feeding 180^o out of phase.

RHC

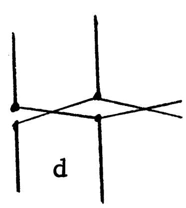


Scaled by $\tau^{\frac{1}{2}}$

ANALYSIS OF LPDA

We have been using a scaling constant, τ , to scale the elements of the antenna. The scaling constant of a log periodic antenna should scale the antenna exactly, but this is not the case. When all the elements are multiplied by τ, the phase of the antenna changes by 180^o. The true scaling constant is τ^2. Some papers in the literature use this constant. The fundamental cell of the antenna contains two dipoles and a cross over.

Feed



Fundamental Cell

It is tempting to think of this as a dipole and a parasitic element, but this is not the proper analysis. Both elements are fed. We can analyze this as a two element array. The extra distance, d, that the signal travels from the feed element to the second decreases the phase until the second element becomes like the reflector on page 318 and gives a backfire radiation. Near the point where the elements are a half wavelength long, the currents will rise on the dipoles because the impedance presented to the transmission line feeder is better matched.

Assume that the active region for a given frequency is near the middle of the antenna. The elements near the feed are mismatched to the feeder transmission line and have small currents on them. Also the elements are alternately phased so that the radiation from one element is cancelled by the near by elements which are out of phase. This region is similiar to the center portion of the spiral antenna or the top portion of the conical spiral. The waves are in the slow wave section of the antenna: a shunt capactively loaded transmission line. When the active region is reached, the currents rise in the elements and the phase shift between elements is sufficient to give a backfire radiation. At this point the waves are transforming from a slow wave region to a fast wave region and radiate. Beyond the active region, the currents attenuate rapidly. Using this simple one mode analysis we cannot see any reason for this attenuation except for the mismatch of the elements to the transmission line. But this will not explain all the attenuation.

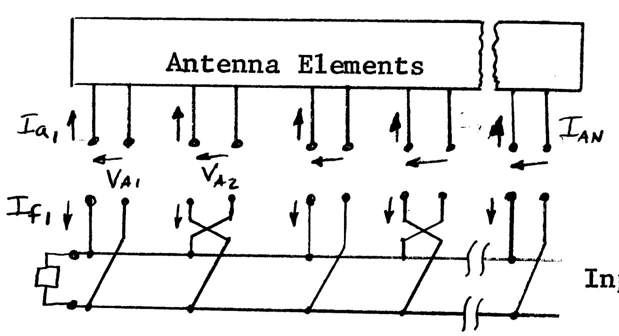
There have been two approaches to a detailed analysis. Carrel uses a circuit approach. The antenna is divided into an antenna section and a feeder section. A mutual impedance matrix can be found for the elements alone and the corresponding admittance matrix which is the matrix inverse. $Z_A = Y_A^{-1}$

$$Y_A = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} & \cdots & Y_{1N} \\ Y_{12} & Y_{22} & Y_{23} & \cdots & \cdot \\ \vdots & & & & \vdots \\ Y_{1N} & & & & Y_{NN} \end{bmatrix}$$

Similiarly we can find the admittance matrix of the feed circuit.

$$Y_F = \begin{bmatrix} Y_{11} & Y_{12} & 0 & \cdots & \cdots & \cdots & 0 \\ Y_{21} & Y_{22} & Y_{23} & 0 & \cdots & \cdots & 0 \\ 0 & Y_{32} & Y_{33} & Y_{34} & 0 & \cdots & 0 \\ \vdots & & & & & & \\ \vdots & & & & & Y_{NN-1} & Y_{NN} \end{bmatrix}$$

Where all terms except the diagonal and the line off the diagonal are zero. The two networks are connected in parallel.



Terminating Z

Input

Feeder Circuit

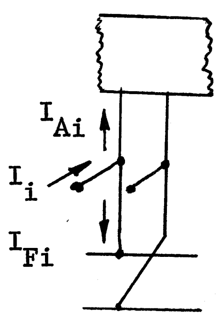
Notice that the feeder circuit includes the phase reversals. The current into the parallel connection of the feed circuit and the antenna is the sum of the two currents into each part. The voltages are the same. These define the node currents and voltages for the combination.

$$I_F = Y_F V_F \qquad\qquad I_A = Y_A V_A$$

$$I = I_F + I_A = (Y_A + Y_F) V_A$$

Multiply through by the matrix Z_A and we get the relation.

$$I = (U + Y_F Z_A) I_A \qquad U \text{ - unit matrix}$$



Antenna Element

Currents on the Connection of the Feed and the Antenna

Feeder Circuit

The driving point current vector, I, has a non-zero term only at the input. The currents in the antenna element bases can be found by inverting the matrix.

$$T = U + Y_F Z_A$$

$$I_A = (U + Y_F Z_A)^{-1} I$$

Once we know the base currents, then the antenna pattern can be calculated and the active region identified. The second approach analyzes the antenna as a tapered periodic structure. The currents are expanded in terms of the space harmonics of the periodic structure. This is a multimode analysis. The current decreases after the active region because it is coupled into higher order space harmonic modes which attenuate rapidly. The first method does not implicitly use the mode method although it is there with the coupling between elements. We could find eigenmodes from the matrix equation. Both the log periodically scaled and the continuously scaled antennas reduction in current after the active region can be explained by mode coupling through the active region of the antenna. The net result is the radiation of the energy in the active region and the reduction of the currents. This reduction enables the truncation of the antenna.

A successful log periodically scaled antenna must have a scaling constant near enough to one so that ripples between exactly scaled frequencies are small enough to be insignificant. Second there must be coupling between the different elements so that higher order modes will be generated. The phasing on these must be so the currents are attenuated after the active region with the structure transforming from a slow wave to a fast wave structure through the active region.

FEEDING THE LPDA

Like the spiral antenna, the truncation principle allows the antenna to be fed with an infinite balun. The currents on the antenna attenuate rapidly after the active region which means any currents which are induced on the antenna beyond the active region toward the low frequency end will not couple into the feed of the antenna. This is one of the consequences of backfire radiation. When we discussed spirals, it was said that there are nulls in the pattern in the direction of increasing structure. It is also true for the log periodically scaled antenna.

The infinite balun consists of a coax line soldered to one side of the antenna feed line. At the feed end the center conductor is jumpered to the other feed line. To maintain symmetry an outer shield of a coax is soldered to the other feeder. The current on the outer conductor of the coax flows out on to the outer surface and down the feeder. The outer shield of the coax and the dummy coax become part of the antenna. The currents on the feeder are reduced from the truncation and backfire radiation. Hence currents induced on the outer shield of the coax beyond the active region will not couple into the feed region. A diagram of the feeder and infinite balun is on page 483.

The input impedance will varying with the same frequency ripple spacing as the pattern response. Carrel has devised a method of calculating the mean impedance of the antenna. The impedance will vary about this value with a peak VSWR of about 2:1. Suppose the feeder has a characteristic impedance of Z_o without the elements. We can relate this to a capacitance per unit length from the expression on page 70.

$$Z_o = \frac{1}{v_o C}$$

Where C is the capacitance per unit length and v_o is the free space velocity of light. The elements of the antenna can be approximated as small capacitances shunted across the line. The impedance of the antenna is approximately the impedance of the feed including the shunt capacitance of the elements. Before the active region is reached, the antenna is a transmission line which acts as a tapered transmission line transformer into the active region. If we divide the capacitance of the elements by a length, we obtain a second capacitance per unit length due to the elements. These two capacitances add to give the total capacitance per unit length of the equivalent transmission line. The overall effect of the elements is to lower the impedance of the feeder.

The capacitance of a small dipole is proportional to its length.

$$Z = -j Z_a \operatorname{Cot}(\beta L_i/2)$$

Where L_i is the total length of the dipole. Z_a is the average characteristic impedance of a dipole.

$$Z_a = 120 \left(\operatorname{Ln}(L_i/(2a)) - 2.25 \right)$$

Where a is the radius of the dipole. We have approximated the dipole as a shunt open circuited stub. We can replace the cotangent by its small argument approximation. The capacitance of the dipole becomes

$$C_i = \frac{L_i}{2\, v_o\, Z_a}$$

The length associated with this capacitance is the geometric mean of the two spacings on either side.

$$d_{mean} = \sqrt{d_n\, d_{n-1}} = \frac{d_n}{\sqrt{\tau}}$$

The capacitance per unit length due to the dipole becomes

$$\Delta C = \frac{C_i}{\text{Length}} = \frac{L_i \sqrt{\tau}}{2\, v_o\, d_i\, Z_a}$$

But from page 461 we have $\sigma = \frac{d_n}{2\, L_n}$. We can substitute this in and eliminate the dependence on lengths.

$$\Delta C = \frac{\sqrt{\tau}}{4\sigma\, v_o\, Z_a}$$

If the ratio of the diameter to the length of the element remains constant throughout the antenna, then the capacitance per unit length is constant through out the antenna. The capacitance due to the lengths of the dipole elements increases as the same ratio as the distances between the elements. This ratio is the scaling constant of the antenna.

The average input resistance is given by

$$R_o = \sqrt{\frac{L_o}{C_o + \Delta C}} \qquad L_o \text{ - inductance per unit length}$$

Substituting C into the equation, we get

$$R_o = \sqrt{\frac{L_o}{C_o + \frac{\sqrt{\tau}}{4\sigma\, v_o\, Z_a}}} = \sqrt{\frac{L_o}{C_o}}\sqrt{\frac{1}{1 + \frac{\sqrt{\tau}}{4\sigma\, v_o C_o Z_a}}}$$

We can make the substitutions

$$Z_o = \sqrt{\frac{L_o}{C_o}} = \frac{1}{v_o\, C_o}$$

$$R_o = Z_o \sqrt{\frac{1}{1 + \frac{\sqrt{\tau}}{4\sigma}\,\frac{Z_o}{Z_a}}}$$

This will give the mean value of resistance at the input. If the antenna is elevated as shown on the bottom of page 476, then the feed line will be an impedance transformer to the higher impedances of the lower frequency end. The initial feed region can become very critical in these cases.

The currents on the feeder will radiate like any other current. These are traveling wave currents while the currents on the elements are standing waves. The standing wave currents are larger and radiate more energy. The two sides of the feeder have currents in opposite directions and their radiation will tend to cancel each other. This radiation from the feeder will limit the cross polarization response. Because these currents on the feeder are traveling wave currents, they will radiate predominately in the forward direction; opposite the backfire radiation of the elements. These traveling wave currents are slowed by the elements which broadens the beam and gives significant radiation in the backfire direction. See pages 359 and 360 for an example of greatly slowed traveling wave radiation. There will still be a null in the direction of the feeder because the element pattern is the incremental dipole. Most antennas have some elevation angle, ψ , which will fill in the null.

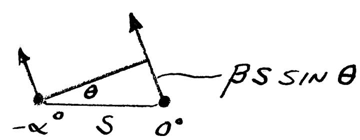
On the diagram on page 483 we see that the jumper at the feed will have a finite length. As the upper frequency bandend is increased this length becomes more significant. The jumper can be respresented as a series inductor at the input of the antenna. It will be a complex network of inductance and capacitance, but for our purpose modeling it as an inductor is sufficient. The inductor will give a phase shift between the two sides of the antenna which will squint the beam. The phase shift of a series inductor in a transmission line is given by

$$\alpha = -\tan^{-1}\left(\frac{\omega L}{2Z_0}\right)$$

For small values of α , Tan α = α which then gives us

$$\alpha = \frac{-180\,\omega L}{2\pi Z_0} \qquad \text{in degrees}$$

We can represent the antenna as a two element array with a phase shift between the elements.



We can find the direction of maximum radiation from the diagram above. Draw a zero reference plane through the phase shifted element and adjust θ until the phase shift to the directly fed element equals α .

$$\beta S \sin\theta = \alpha \qquad \theta = \sin^{-1}\left(\frac{\alpha}{\beta S}\right)$$

The beam will be squinted in the direction of the jumpered element and will increase with frequency.

LOG PERIODIC ANTENNA TYPES

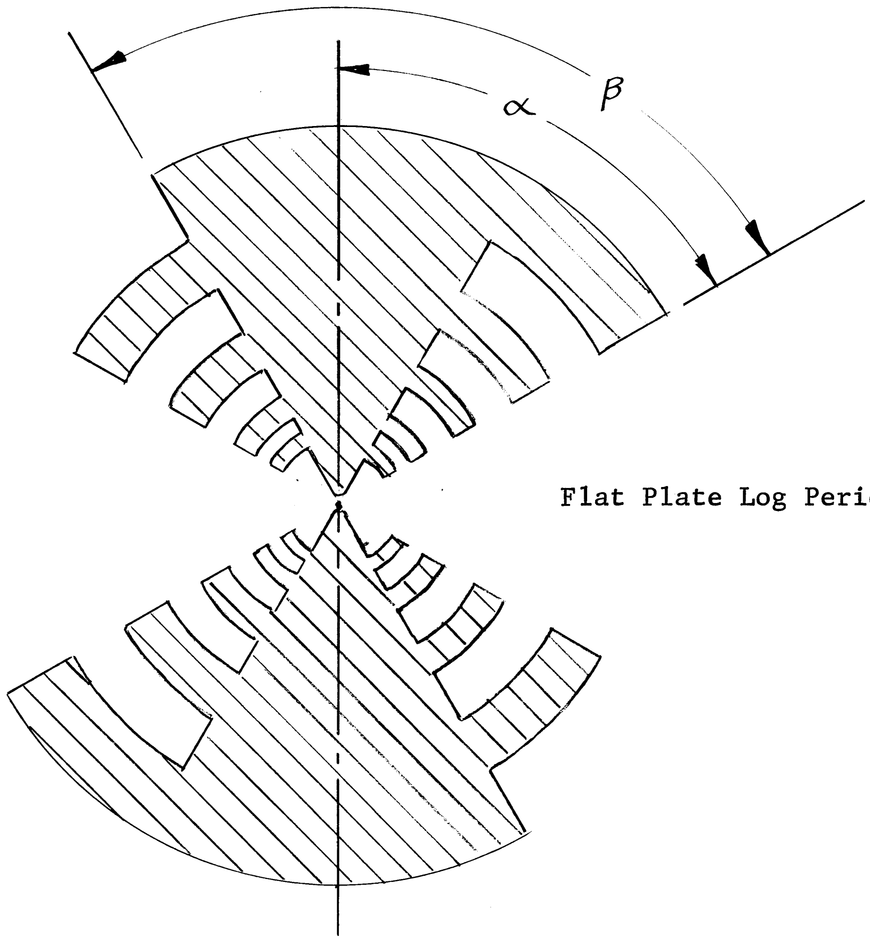
The other log periodic antennas we will look at have similiar properties to the log periodic dipole antenna (LPDA). They are linearly polarized with a significant cross polarization response. We will describe the antennas using the same parameters as the LPDA even though the literature has different paramters in some cases. Usually the literature uses the true scaling constant, see page 479, and the total angle between the sides is used instead of ψ defined above as being between the center plane and the arm.

The first log periodic antennas were cut from flat sheets by R. H. DuHamel and D. E. Isbell of the University of Illinois as shown on the top of page 487. The patterns are the same on both side of the antenna which we can see from symmetry. The pattern is polarized in the direction of the teeth which shows that the resonant currents on the teeth are much larger than the traveling wave radial currents. The angle α on the figure corresponds to the half apex angle of the LPDA. The other angle β describes the width of the bifin portion of the antenna.

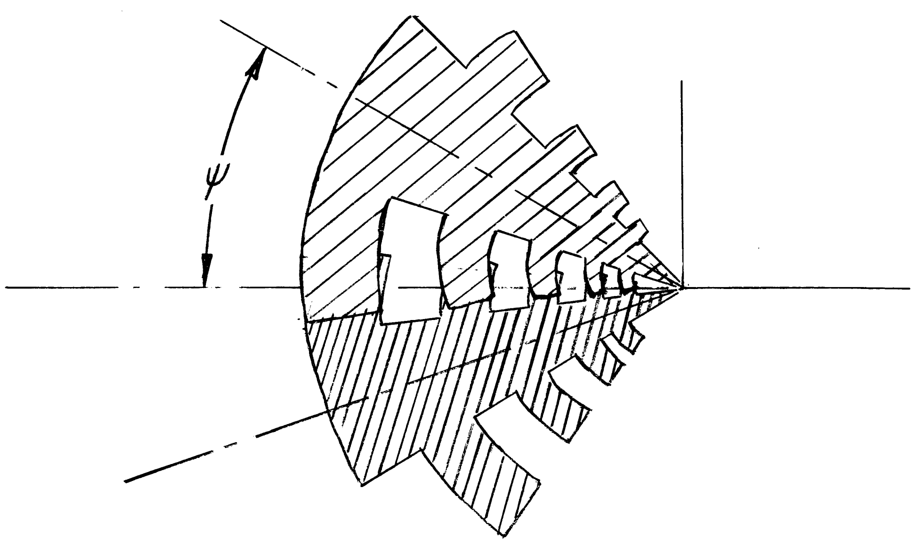
Isbell took the design above and inclined the two sides as shown on the bottom of page 487. This reduced the pattern backlobe response to give a undirectional pattern. The antenna above has an impedance of about 189 ohms for a complementary structure. Bending the two arms together lowers the input impedance which is reported to be 70 ohms when the elevation angle, ψ, is 15°. Finually DuHamel and F. R. Ore straighten the teeth to produce the trapezoidal tooth log periodic antenna which is shown on page 488. In this design the width of the teeth equals the spacing between them. We still have the same definition of the half apex angle α and the width of the bifin is defined by the angle β. The ratio of the lengths R_1 and R_2 is given by a scale factor, τ. Again this conforms to LPDA definition and is the square root of the real scaling constant. The two sides are separated by the angle 2ψ and the sides project to a virtual apex. This antenna has a good structure for high frequency operation because it is largely self supporting. It can be fed from an infinite balun by soldering a coax and a dummy coax on the two arms. It is not necessary to have the teeth the same width as the spacing. Reducing the teeth width logically transforms the antenna into the LPDA. Note that the distances R_i are always measured from the bottom of the teeth.

The trapezoidal tooth log periodic antenna is impractical for low frequency antennas because of the size and weight on the teeth. DuHamel and Ore tried making the antenna with a wire outline which followed the outside dimensions of the teeth. This antenna has practically the same pattern as the trapezoidal tooth antenna but reduces the weight and wind loading to produce a practical low frequency antenna. A diagram of this antenna is given on page 489. Notice that the direction of the teeth alternates on the two sides of the antenna. A table of designs found by DuHamel and Ore is given on page 490. It was found experimentally that the spacing constant, σ, is limited to less than or equal to 0.3; for larger spacings the antenna pattern breaks up. One of the advantages of the trapezoidal tooth antenna is that smaller scaling constants can be used compared to the LPDA. The log periodic dipole antenna (LPDA) must have a scaling constant greater than or equal to 0.8, but the LPTT antenna has good patterns even with $\tau = 0.63$. This is possible because their is greater coupling between the flat plates of the teeth compared to dipoles. As the width of the teeth is reduced the lowest possible scaling constant increases until the antenna is a LPDA.

The shape of the teeth is not too important as long as they scale. On page 491 is a diagram of a triangular tooth antenna. The distances from the



Flat Plate Log Periodic Antenna



virtual apex to the ends of the triangles, R_i, scale by the scaling constant. The antenna can also be made from a wire outline structure as shown on page 492. This antenna has an easier construction than the trapezoidal tooth wire outline antenna. Its patterns are similiar to the trapezoidal tooth antenna although the coupling between teeth will be less and will have a larger scaling constant lower bound than the trapezoidal tooth antenna.

Successful Trapezoidal Tooth Outline Antennas

Scaling Constant	Elevation Angle	Half Apex Angle	E Plane Beamwidth	H Plane Beamwidth	Directivity	Side-lobes
0.63	15	30	85	153	5	12
0.63	15	37.5	74	155	5.6	12.4
0.71	15	30	70	118	7.	17.7
0.71	15	37.5	66	126	7	17
0.63	22.5	30	86	112	6.3	8.6
0.63	22.5	37.5	72	125	6.6	11.4
0.71	22.5	30	71	95	7.9	14
0.71	22.5	37.5	67	106	7.6	14.9
0.77	22.5	30	67	85	8.6	15.8
0.84	22.5	22.5	66	66	9.8	12.3
0.84	22.5	30	64	79	9.1	15.8
0.63	30	30	87	87	7.4	7
0.63	30	37.5	73	103	7.4	8.6
0.71	30	30	71	77	8.8	9.9
0.71	30	37.5	68	93	8.1	12.8

To end this discussion of log periodics we will discuss a use from another field. The musical scale was originally based on the Pythagorean system which gives the intervals between notes as the ratio of integers. This system works fine for melodies using only the diatonic scale, but when chromatic notes are introduced, the system proves to be inadequate. It became necessary to devise other methods of tuning because the system gave perfect pitches for simple notes, but intolerably wrong pitches for some notes. In fact instruments were tuned to particular keys. By the 16th century organs were being made with split keys so that all different melodies could be played. Different parts of the keys controlled different pipes.

There were a number of solutions to this problem but the one which survives is the even tempered scale. This scale spreads the inaccuracies over all the intervals. There are 12 half steps or semitones between octaves (the doubling of frequency). Even tempered means an equal ratio between notes or log periodic. We can find the scaling constant of this system.

$$\tau = \frac{1}{2^{1/12}} = 0.944$$

If we take instruments such as organ pipes whose resonant frequencies are determined by lengths, we can find the ratio of the lengths by using the scaling constant.

