RADIATION FROM CURRENTS

When we study the radiation from currents in wires, we will combine the ideas of transmission lines and arrays. First, we must find the radiation from a current element. The easiest way to do this is to use the magnetic vector potential, which is a function of the currents and their distributions in space. The magnetic vector potential can be used to find both the near field and the far field, but we will simplify it until it gives only the latter,

MAGNETIC VECTOR POTENTIAL

Take the two curl equations of Maxwell in phasor notation.

$$\nabla \times \bar{E} = -j\omega\mu \bar{H} \qquad \nabla \times \bar{H} = j\omega\epsilon \bar{E} + \bar{J}$$

From the equation for the divergence of the magnetic field

$$\nabla \cdot \bar{H} = 0$$

we can say that the magnetic field is the curl of some vector function. In general a vector field can be expressed as the gradient of a scalar function plus the curl of some vector function. We will express the magnetic field as the curl of a vector function.

$$\bar{H} = \nabla \times \bar{A}$$

(Some texts use $\bar{B} = \nabla \times \bar{A}$)

If we substitute this in the first equation, we can combine the two curls.

$$\nabla \times \bar{E} = -j\omega\mu(\nabla \times \bar{A})$$

$$\nabla \times (\bar{E} + j\omega\mu\bar{A}) = 0$$

A vector whose curl is zero can be derived from the gradient of a scalar function.

$$\bar{E} + j\omega\mu\bar{A} = -\nabla\phi$$

ϕ is called the electric scalar potential. Now substitute A in the second curl equation.

$$\nabla \times \nabla \times \bar{A} = j\omega\epsilon\bar{E} + \bar{J}$$

$$\bar{E} = -\nabla\phi - j\omega\mu\bar{A}$$

Substituting the equation for the electric field, we can eliminate it from the equation of the magnetic vector potential, A.

$$\nabla\times\nabla\times\bar{A} - \omega^2\mu\epsilon\bar{A} = \bar{J} - j\omega\epsilon\nabla\phi$$

The term curl curl can be expanded to

$$\nabla\times\nabla\times\bar{A} = \nabla(\nabla\cdot\bar{A}) - \nabla^2\bar{A}$$

The equation becomes

$$\nabla(\nabla\cdot\bar{A}) - \nabla^2\bar{A} - \omega^2\mu\epsilon\bar{A} = \bar{J} - j\omega\epsilon\nabla\phi$$

There is an arbitrariness in the vector field $\bar{A}$. We have picked the curl $\bar{A}$ and to define $\bar{A}$ completely $\nabla\cdot\bar{A}$ must be picked. The traditional choice is to let

$$\nabla\cdot\bar{A} = -j\omega\epsilon\phi$$

Then the equation of A simplifies to

$$\nabla^2\bar{A} + \omega^2\mu\epsilon\bar{A} = -\bar{J}$$

We can identify $\omega^2\mu\epsilon$ as β^2. The electric and magnetic fields can be found from the magnetic vector potential by

$$\bar{E} = -j\omega\mu\bar{A} + \frac{1}{j\omega\epsilon}\nabla(\nabla\cdot\bar{A})$$

$$\bar{H} = \nabla\times\bar{A}$$

We have reduced the problem to solving a single partial differential equation. We can simplify this more if we only consider the static case. From the Biot Savart relation we can find the magnetic field from

$$d\bar{H} = \frac{\bar{J}dV'\times(\bar{r}-\bar{r}')}{4\pi|\bar{r}-\bar{r}'|^3}$$

where $\bar{J}'$ is the current density, $\bar{r}$ the field coordinate, $\bar{r}'$ the source coordinate, and dV' the differential volume of the source. The cross product of the vectors can be expressed

$$\frac{\bar{J}dV'\times(\bar{r}-\bar{r}')}{|\bar{r}-\bar{r}'|^3} = \nabla\left(\frac{1}{|\bar{r}-\bar{r}'|}\right)\times\bar{J}'dV'$$

This can be expanded to

$$\nabla\left(\frac{1}{|\bar{r}-\bar{r}'|}\right)\times\bar{J}'dV' = \nabla\times\left(\frac{\bar{J}dV'}{|\bar{r}-\bar{r}'|}\right) - \frac{1}{|\bar{r}-\bar{r}'|}\nabla\times\bar{J}'dV'$$

The last term is zero because the del operator is on field coordinates and $J'\,dV'$ is in source coordinates. The Biot Savart relation becomes

$$d\bar{H} = \nabla\times\left(\frac{\bar{J}'dV'}{4\pi|\bar{r}-\bar{r}'|}\right)$$

The total magnetic field is the integral of this

$$\bar{H} = \nabla\times\iiint\frac{J'dV'}{4\pi|\bar{r}-\bar{r}'|}$$

We can identify the integral as the magnetic vector potential.

$$\bar{A} = \iiint \frac{\bar{J}\, dv'}{4\pi |\bar{r} - \bar{r}'|}$$

Keep in mind that this was derived for the static case.

Now we want to consider a current element I' dl' at the origin directed in the Z direction. Since there is only a Z component of current, the boundary conditions will leave us with only a Z component of the magnetic vector potential. In any region away from the origin the partial differential equation of $\bar{A}$ reduces to

$$\nabla^2 A_z + \beta^2 A_z = 0$$

We expect that the vector field, A, will be spherically symmetric since the only source is a point at the origin. We will take only the R component of the Laplacian operator and are left with the following differential equation.

$$\frac{1}{r^2}\left(\frac{d}{dr} r^2 \frac{dA_z}{dr}\right) + \beta^2 A_z = 0$$

The two independent solutions to this second order differential equation are

$$\frac{1}{r} e^{-j\beta r} \qquad \frac{1}{r} e^{j\beta r}$$

These are two waves which are traveling away and toward the origin. They are the spherical waves. We will consider only the outward traveling wave which gives the solution.

$$A_z = \frac{c}{r} e^{-j\beta r}$$

We can evaluate the constant by considering the static case when $\beta \rightarrow 0$. We have a solution for this in the integral above. The integral over a delta function of the current component at the origin gives just the value of the current element.

$$A_z = \iiint \frac{\bar{J}' \delta(v)\, dv'}{4\pi |\bar{r} - \bar{r}'|} = \frac{I'\ell'}{4\pi r}$$

We can now identify the solution as

$$A_z = \frac{I\ell}{4\pi r} e^{-j\beta r}$$

We could do this for every point in a current distribution which would give us the equation for the magnetic vector potential when we assume spherical waves. The result is a modified integral equation for the vector potential.

$$\bar{A} = \iiint \frac{\bar{J}' e^{-j\beta |\bar{r} - \bar{r}'|}\, dv'}{4\pi |\bar{r} - \bar{r}'|}$$

We can recognize this as the retarded magnetic vector potential.

We can use the magnetic vector potential for the current element to find the solution for an incremental dipole antenna. The potential is

$$A_z = \frac{I\ell}{4\pi r} e^{-j\beta r}$$

The electric and magnetic fields can be found from the equations given above

$$\bar{E} = -j\omega\mu\bar{A} + \frac{1}{j\omega\epsilon}\nabla(\nabla\cdot\bar{A})$$

$$\bar{H} = \nabla\times\bar{A}$$

The result is the fields around the antenna. When we convert the Z component of the electric field to a θ component and do the necessary derivatives, the following equations are obtained.

$$E_r = \frac{I\ell}{2\pi} e^{-j\beta r}\left(\frac{\eta}{r^2} + \frac{1}{j\omega\epsilon r^3}\right)\cos\theta$$

$$E_\theta = \frac{I\ell}{4\pi} e^{-j\beta r}\left(\frac{j\omega\mu}{r} + \frac{\eta}{r^2} + \frac{1}{j\omega\epsilon r^3}\right)\sin\theta$$

$$H_\phi = \frac{I\ell}{4\pi} e^{-j\beta r}\left(\frac{j\beta}{r} + \frac{1}{r^2}\right)\sin\theta$$

Above is the total field of the incremental dipole valid at all frequencies and distances from the dipole. η is the intrinsic impedance of free space.

We divide these into two groups. The terms with 1/R dependence are the far field radiation terms. All those with $1/R^2$ or $1/R^3$ are called the induction fields. The far field contains only E_θ and H_ϕ components. The far field electric field is

$$E_\theta = \frac{I\ell}{4\pi} e^{-j\beta r}\, \frac{j\omega\mu}{r}\sin\theta$$

We evaluate ω as $2\pi F$ and split μ into $\sqrt{\mu}\,\sqrt{\mu}$ and divide and multiply by $\sqrt{\epsilon}$.

$$E_\theta = j\frac{I\ell\, 2\pi F\sqrt{\mu\epsilon}}{4\pi r}\sqrt{\frac{\mu}{\epsilon}}\, e^{-j\beta r}\sin\theta$$

The following terms can be recognized as

$$c = \frac{1}{\sqrt{\mu\epsilon}} \qquad \frac{F}{c} = \frac{1}{\lambda} \qquad \eta = \sqrt{\frac{\mu}{\epsilon}}$$

The θ component of the electric field becomes

$$E_\theta = \frac{jI\ell\eta}{2\lambda r} e^{-j\beta r}\sin\theta$$

The magnetic field can be found from the electric field.

$$H_\phi = E_\theta / \eta$$

$$H_\phi = \frac{jI\ell}{2\lambda r} e^{-j\beta r} \sin\theta$$

These are the far field radiation terms of the incremental dipole.

The far field of a current distribution can be derived from the magnetic vector potential if it is simplified. The electric field in the far field is given by

$$\bar{E} = -j\omega\mu\bar{A}$$

where for the far field A becomes

$$\bar{A} = \iiint \frac{\bar{J}' e^{-j\beta|\bar{r}-\bar{r}'|}}{4\pi r} dv'$$

In most cases the volume integral reduces to a single coordinate integral which is the center of the wire carrying the current. The far field approximation $|\bar{r}| \simeq |\bar{r} - \bar{r}'|$ has been made for amplitudes but not for phases. We will pick any convenient plane defined by the direction of radiation to be the zero reference plane. The magnetic field can be found from the electric field in a propagating wave.

We can illustrate this by working out the incremental dipole. The current density is assumed to be a Dirac delta function at r = 0 times the current moment, I 1. The magnetic vector potential becomes

$$A_z = \iiint_v \frac{I\ell\,\delta(\bar{r}')\, e^{-j\beta|\bar{r}-\bar{r}'|}}{4\pi r}$$

$$A_z = \frac{I\ell}{4\pi r} e^{-j\beta r}$$

Where r is the magnitude of $\bar{r}$. The electric field is $\bar{E} = -j\omega\mu\bar{A}$.

$$\bar{E} = -j\omega\mu \frac{I\ell}{4\pi r} e^{-j\beta r}\bar{a}_z$$

There is only a E_z component in the same direction as A_z. $\bar{a}_z$ is the unit vector in the Z direction. In the far field these waves are spherical. We need to find the θ and ϕ components of the electric field. The θ component is the projection of the electric field on the $\bar{a}_\theta$ vector (vector dot product).

$$E_\theta = -j\omega\mu \frac{I\ell}{4\pi r} e^{-j\beta r}\, \bar{a}_z \cdot \bar{a}_\theta$$

$$\bar{a}_z \cdot \bar{a}_\theta = -\sin\theta$$

$$E_\theta = j\omega\mu \frac{I\ell}{4\pi r} e^{-j\beta r} \sin\theta$$

This is the same expression for the far field given on page 123. The E_ϕ component depends on the dot product: $\bar{a}_z \cdot \bar{a}_\phi = 0$. Therefore $E_\phi = 0$. In general the outward spherical wave will have both E_θ and E_ϕ components from a current distribution. The magnetic field can be found from the electric field as for any plane or spherical wave.

The power in the radiated field can be found by integrating the Poynting vector over a sphere centered on the current element.

$$\bar{S} = \bar{E} \times \bar{H}^* \text{ (assuming RMS values of E \& H)}$$

$$P_r = \iint \bar{E} \times H^* \cdot d\bar{S} = \int_0^{2\pi}\int_0^{\pi} R^2 \sin\theta \, E_\theta H_\phi^* \, d\theta \, d\phi$$

$$P_r = 2\pi \left(\frac{I\ell}{2\lambda}\right)^2 \eta \int_0^{\pi} \sin^3\theta \, d\theta$$

$$P_r = \frac{2\pi}{3}\left(\frac{I\ell}{\lambda}\right)^2 \eta$$

The radiation resistance can be found from the radiated power.

$$R_r = \frac{P_r}{I^2}$$

Therefore the radiation resistance is

$$R_r = \frac{2\pi\eta}{3}\left(\frac{\ell}{\lambda}\right)^2$$

The assumption has been made that the current is uniform over the dipole element which limits the range over which the equation is valid. More limiting is the assumption that the radiation is from a single point, $\ell \ll \lambda$.

STANDING WAVES

A wire antenna can be thought of as a diverging transmission line. Before we discuss the radiation from such a structure, we must find the current distribution on the wire.

Consider an open circuited transmission line. The equations for the voltage and current measured from the load are

$$V = V_0 \left(e^{j\beta s} + \rho e^{-j\beta s}\right)$$

$$I = I_0 \left(e^{j\beta s} - \rho e^{-j\beta s}\right)$$

For an open circuit the voltage reflection coefficient, ρ, equals one and the equations become

$$V = V_0 (e^{j\beta s} + e^{-j\beta s})$$

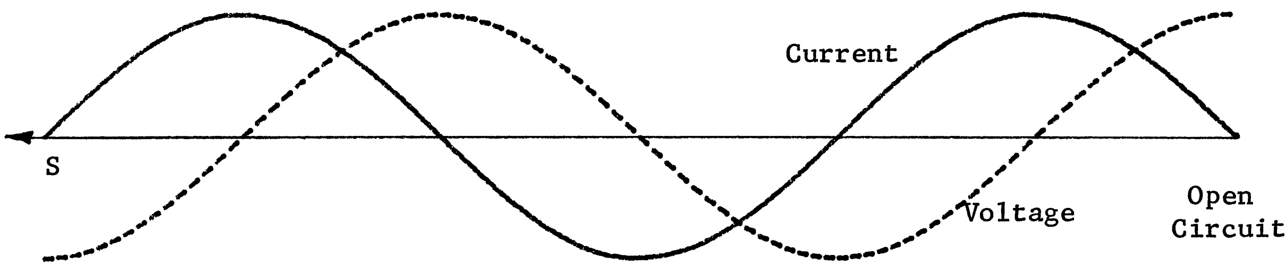
$$I = \frac{V_0}{Z_0} (e^{j\beta s} - e^{-j\beta s})$$

Using the Euler identities, these become

$$V = 2V_0 \cos \beta s$$

$$I = j\frac{2V_0}{Z_0} \sin \beta s$$

Note the current is 90^o out of phase with respect to the voltage and there is no real power transfer since the product of the current and voltage has no real part. The current and voltage are 90^o out of phase in space as well as time. The current is zero for S = 0 which is at the ends of the transmission line. The voltage is a maximum when the current is a minimum and vice versa. See figure.



Standing Wave

SINUSOIDAL CURRENT DISTRIBUTION

When we assume that the radiation of power does not change the current distribution on the wire antenna, we can find the expression for the current on the antenna. Even though it does change the distribution, the effect is only second order to the radiated fields. If the wire is fed in the center, the current distribution is

$$I = I_o \text{ Sin } \beta(\tfrac{L}{2} - |Z|) \qquad \text{or}$$

$$I = I_o \text{ Sin } \beta(\tfrac{L}{2} - Z) \qquad Z \geq 0$$

$$= I_o \text{ Sin } \beta(\tfrac{L}{2} + Z) \qquad Z \leq 0$$

where L is the length of the antenna. This function is zero for $Z = \pm L/2$, the open circuit at the ends of the antenna. We have assumed that there is a standing wave on the antenna. To find the radiated fields we analyze this as a linear continuous array. Given this assumed current density along the Z axis, we find the far portion of the magnetic vector potential.

$$A_z = \frac{I_0 e^{-j\beta r}}{4\pi r}\int_0^{L/2} \sin\beta\left(\frac{L}{2}-z\right) e^{j\beta z\cos\theta}\,dz$$

$$+\frac{I_0 e^{-j\beta r}}{4\pi r}\int_{-\frac{L}{2}}^{0} \sin\beta\left(\frac{L}{2}+z\right) e^{j\beta z\cos\theta}\,dz$$

The result of this integral is

$$A_z = \frac{I_0 e^{-j\beta r}}{2\pi r}\,\frac{\left[\cos\left(\frac{\beta L}{2}\cos\theta\right)-\cos\left(\frac{\beta L}{2}\right)\right]}{\beta\sin^2\theta}$$

We can find the far field electric field from this using: $\bar{E} = -j\omega\mu\bar{A}$

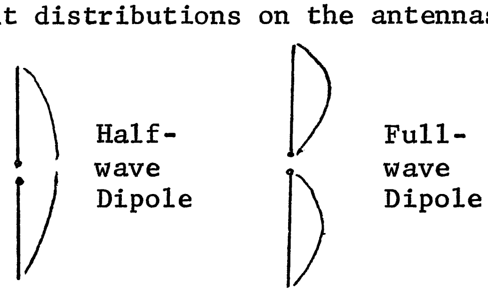
$$E_\theta = \frac{j\omega\mu I_0 e^{-j\beta r}}{2\pi r\beta}\,\frac{\left[\cos\left(\frac{\beta L}{2}\cos\theta\right)-\cos\left(\frac{\beta L}{2}\right)\right]}{\sin\theta}$$

We can reduce this by noting: $\beta = \omega\sqrt{\mu\epsilon}$ and $\eta = \sqrt{\frac{\mu}{\epsilon}}$

$$E_\theta = j\eta\frac{I_0}{2\pi r}e^{-j\beta r}\,\frac{\left[\cos\left(\frac{\beta L}{2}\cos\theta\right)-\cos\left(\beta\frac{L}{2}\right)\right]}{\sin\theta}$$

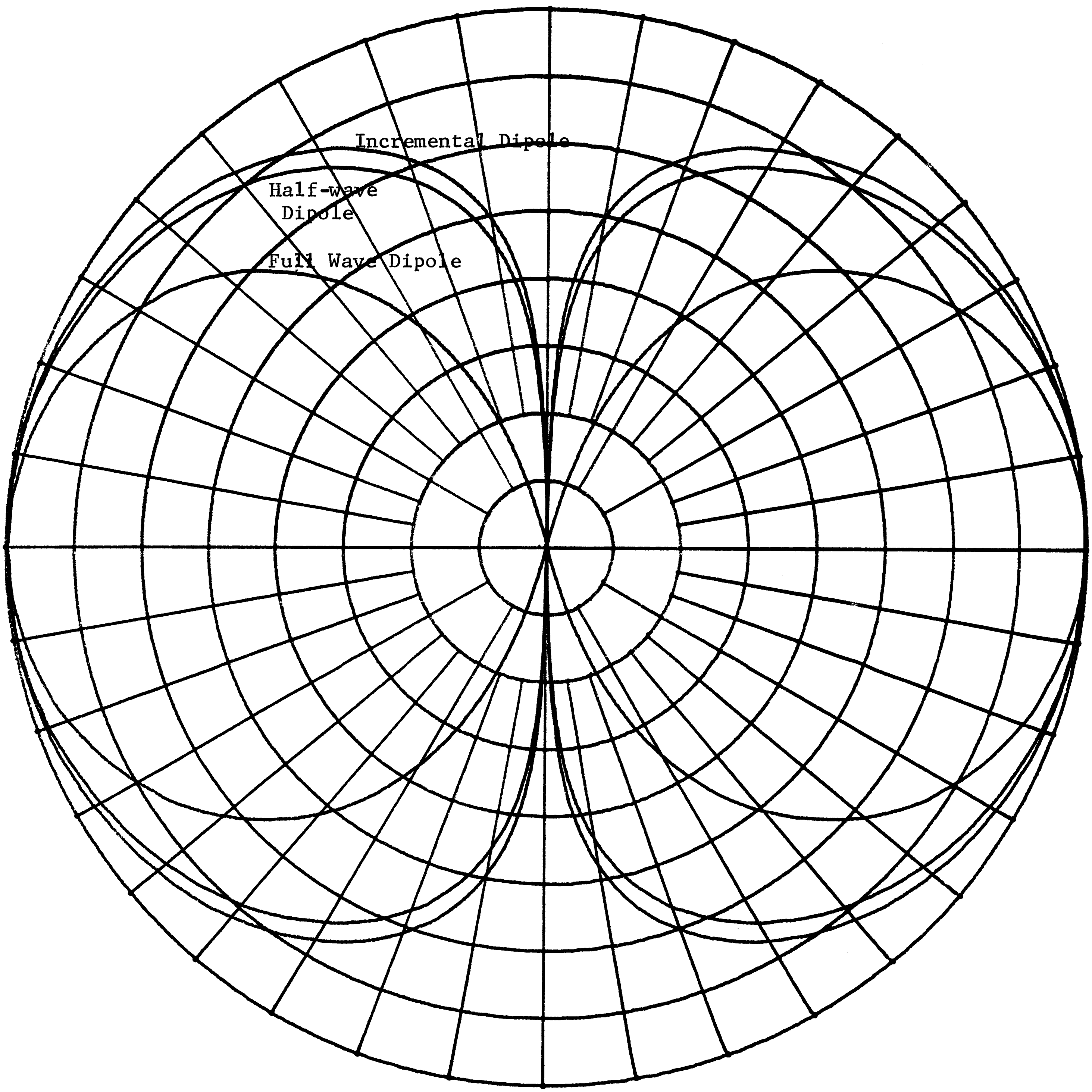
The magnetic field of the spherical wave is found from: $H_\phi = \frac{E_\theta}{\eta}$

The field is linearly polarized since there is only a E_θ component. From symmetry we can see that all the great circle cuts are identical. On page 128 the patterns of the incremental dipole, the half wavelength dipole, and the full wave dipole are plotted on top of each other. We can see that there is only a little difference between the incremental dipole and the half wave dipole. We can understand the patterns if we look at the current distributions on the antennas. These are given in the figure below. In each case the current vanishes at the end of the wires. For each side the current increases from the end in a sinusoidal function. If we consider a line parallel to the Z axis, then the fields from each part of the antenna adds in phase. The theta component of the electric field is related to the Z component of the electric field by the sine of the angle theta. We see from the magnetic vector potential that there is only a Z component of the electric field in the far field. Since Sin(0) = 0, there is a null on the Z axis.



Current Distributions

Comparison of Dipole Patterns



In all cases the dipole will have a null in the pattern at theta = 0. On page 130 there are plots of the dipole at 1.25, 1.5, 1.75, and 2 wavelengths long. The sinusoidal current distributions are given on each pattern. It can be seen from the patterns that there is a null in the pattern at theta equals 90 degrees as the length approaches two wavelengths. If we draw a line parallel to the Z axis and add the fields due to each part of the antenna, then at a length of two wavelengths we can see that there are equal positive and negative portions of current on the dipole.

RADIATED POWER AND RADIATION RESISTANCE

The radiated power density is found from the product of the electric and magnetic field in the far field.

$$S_r = E_\theta H_\phi^* = \frac{\eta |I_0|^2}{(2\pi r)^2}\left[\frac{\cos(\beta\frac{L}{2}\cos\theta) - \cos(\beta\frac{L}{2})}{\sin\theta}\right]^2$$

The total power radiated is the integral of S_r over a large sphere.

$$P_r = \int_0^{2\pi}\int_0^{\pi} S_r r^2 \sin\theta \, d\theta \, d\phi$$

$$= \frac{\eta |I_0|^2}{(2\pi)^2}\int_0^{2\pi}\int_0^{\pi}\frac{\left[\cos(\beta\frac{L}{2}\cos\theta) - \cos(\beta\frac{L}{2})\right]^2}{\sin\theta} d\theta \, d\phi$$

The radiation resistance of the antenna is found from the radiated power.

$$R_r = \frac{P_r}{|I|^2}$$

If we pick the maximum current of the sinusoidal response, I_o, as the reference current, then the radiation resistance is given by

$$R_r = \frac{\eta}{2\pi}\int_0^{\pi}\frac{\left[\cos(\beta\frac{L}{2}\cos\theta) - \cos(\beta\frac{L}{2})\right]^2}{\sin\theta} d\theta$$

The integral can be solved in terms of tabulated functions, the sine and cosine integrals. The radiation resistance referenced to the maximum sinusoidal current is given on page 131.

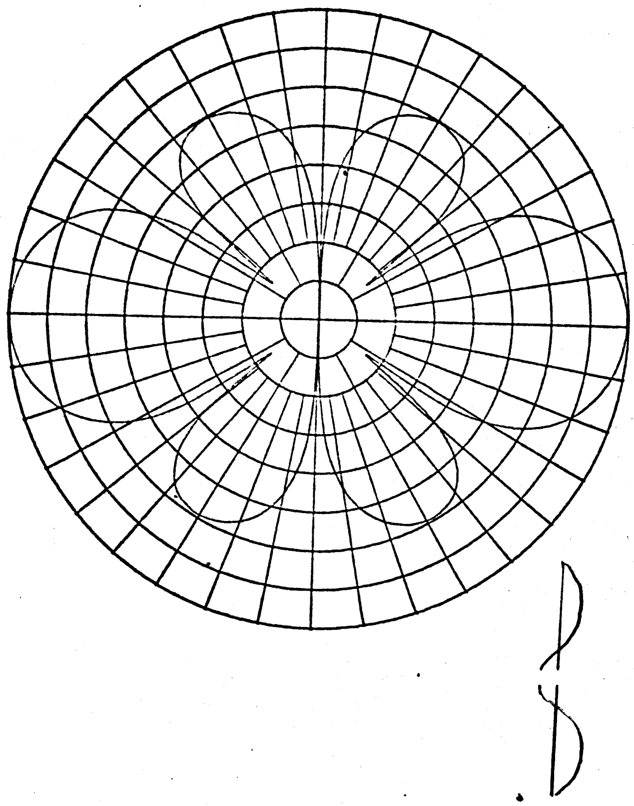
The input impedance is the impedance seen by the source. The radiation resistance is referenced to the current maximum. Input resistance is the ratio of the power radiated to the input current.

$$R_i = \frac{P_r}{|I_i|^2}$$

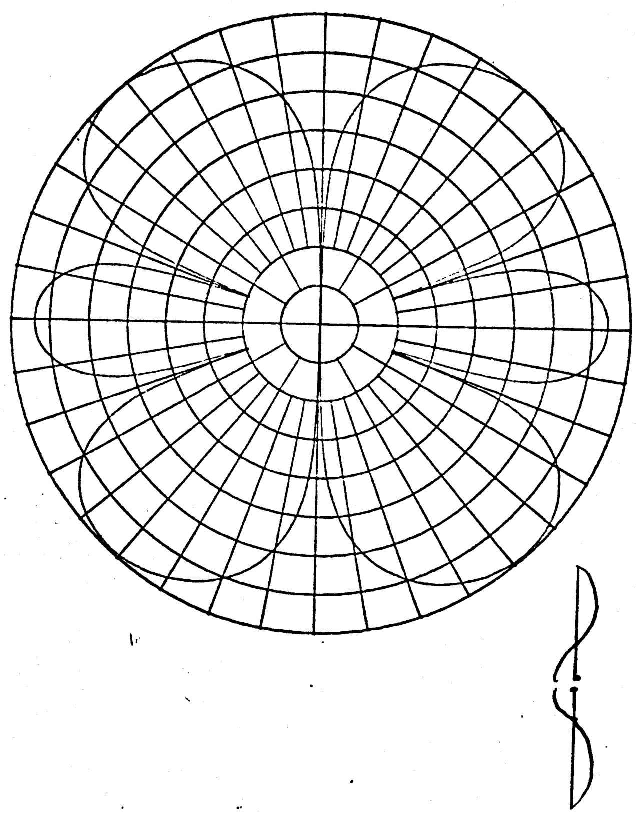
The input current is found from the assumption of a sinusoidal current wave on the antenna. The input current is

$$I_i = I_0 \sin\frac{\beta L}{2}$$

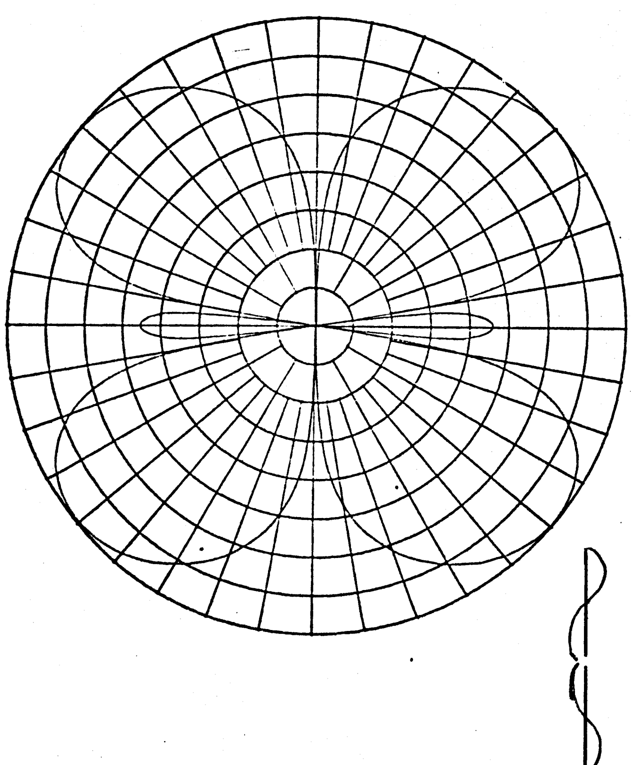
1.25 WAVELENGTHS LONG DIPOLE



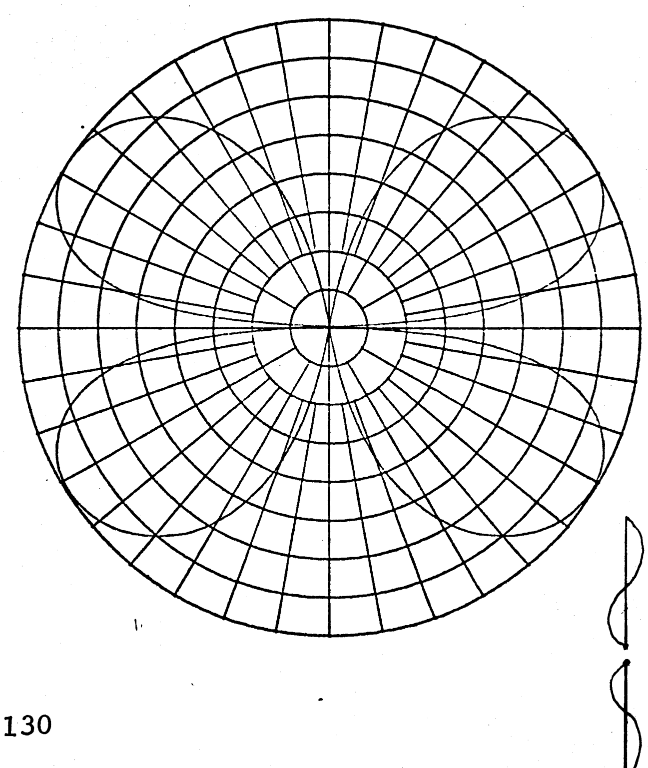
1.5 WAVELENGTHS LONG DIPOLE

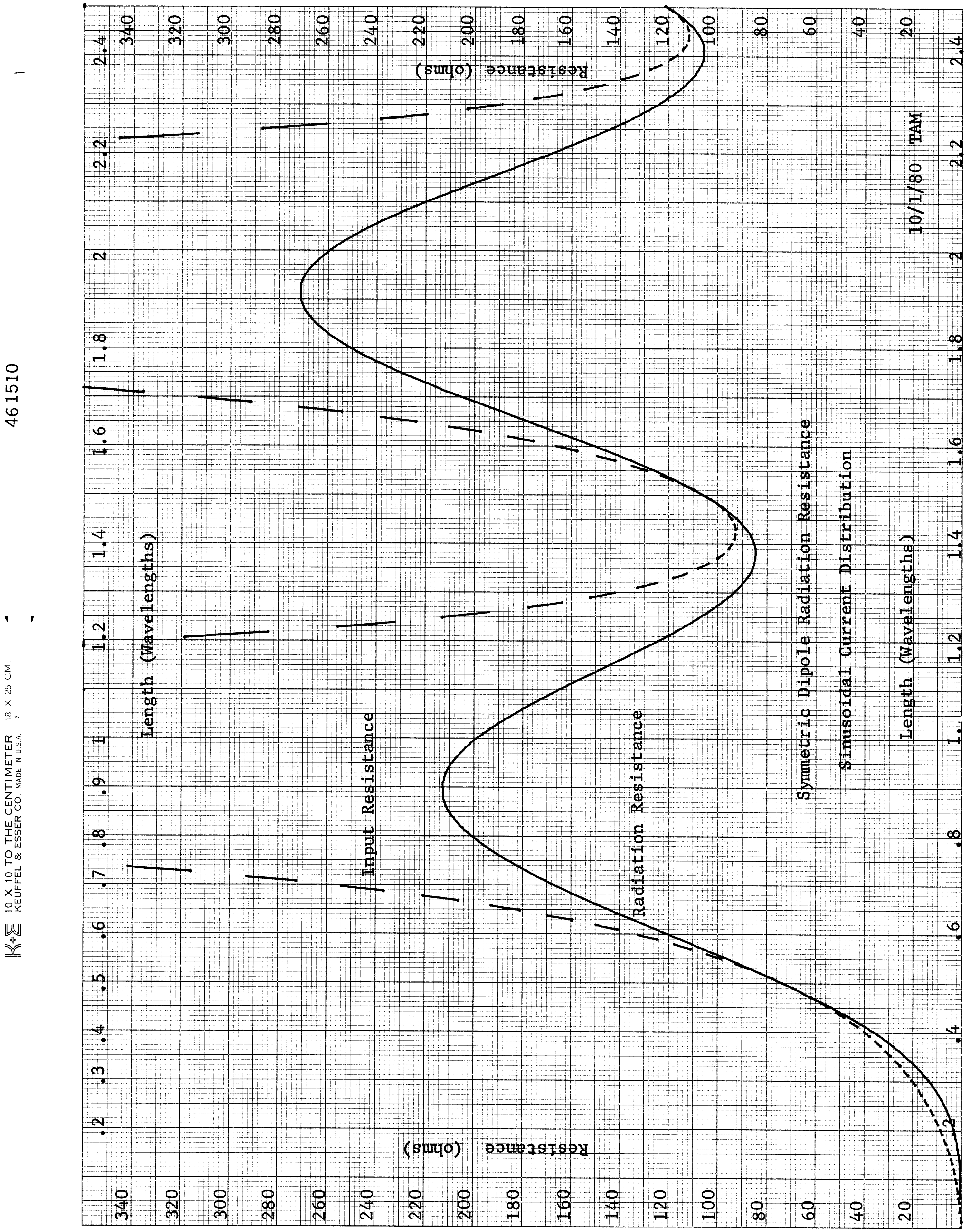


1.75 WAVELENGTHS LONG DIPOLE



2 WAVELENGTHS LONG DIPOLE




Symmetric Dipole Radiation Resistance
Sinusoidal Current Distribution
Length (Wavelengths)
Resistance (ohms)
Input Resistance
Radiation Resistance
10/1/80 TAM
461510
10 X 10 TO THE CENTIMETER 18 X 25 CM.
KEUFFEL & ESSER CO. MADE IN U.S.A.

Using this expression for the input current, the expression for the input resistance is related to the radiation resistance referenced to the maximum current of the sinusoidal current wave.

$$R_i = \frac{R_r}{\sin^2 \frac{\beta L}{2}}$$

The input resistance is plotted as a dashed curve on the plot on page 131. As the length of the dipole approaches one wavelength, the input resistance becomes very large. The actual input resistance depends on the diameter of the wire. Remember this is specifically for the symmetrically fed dipole.

The total input impedance can be found by using the induced emf method of P. S. Carter (1932). This method gives both the input resistance and reactance. On page 133 is a curve of the input impedance of a dipole. The input resistance depends very little on the diameter of the antenna, but the reactance depends on it. The reactance is given for diameters of 0.02, 0.04, and 0.08 wavelengths. Many more curves can be found in Jasik, <u>Antenna Engineering Handbook</u>. The interesting point is around 0.45 wavelengths. The reactance goes to zero. Usually the "half-wave" dipole antenna is made shorter than a half wavelength to resonate out the reactance. For microwave antennas incidental structures of the antenna tend to mask these results. These curves only serve as a guide line.

GAIN (DIRECTIVITY)

We have calculated the total power radiated by the antenna

$$P_r = R_r |I_0|^2$$

For lengths less than a wavelength the maximum of the pattern occurs at theta equals 90 degrees. The directivity of the antenna is found from

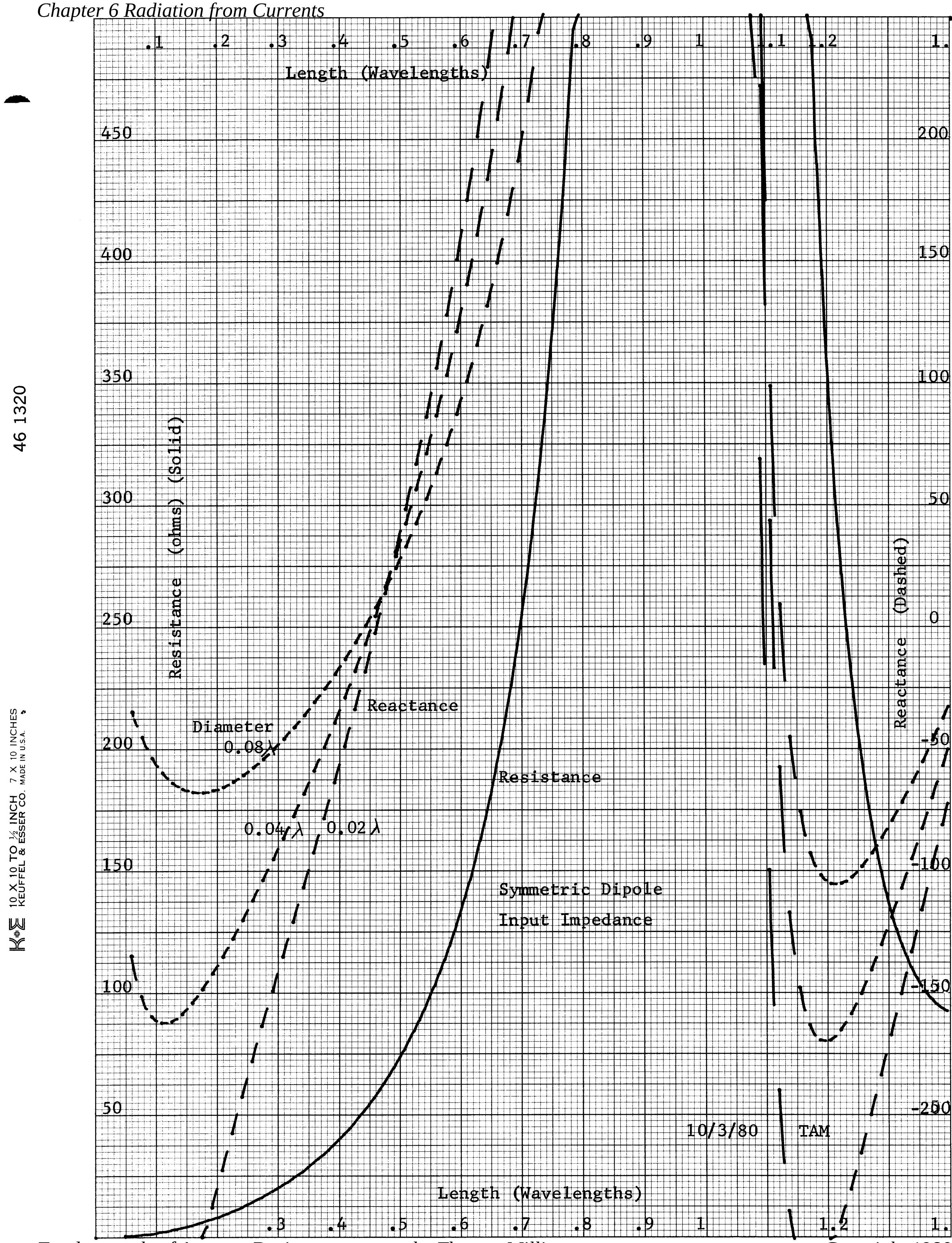
$$\text{Directivity} = \frac{4\pi U_{MAX}}{P_r}$$

$$U_{MAX} = \frac{\eta |I_0|^2}{(2\pi)^2} \left[1 - \cos\left(\beta \frac{L}{2}\right)\right]^2$$

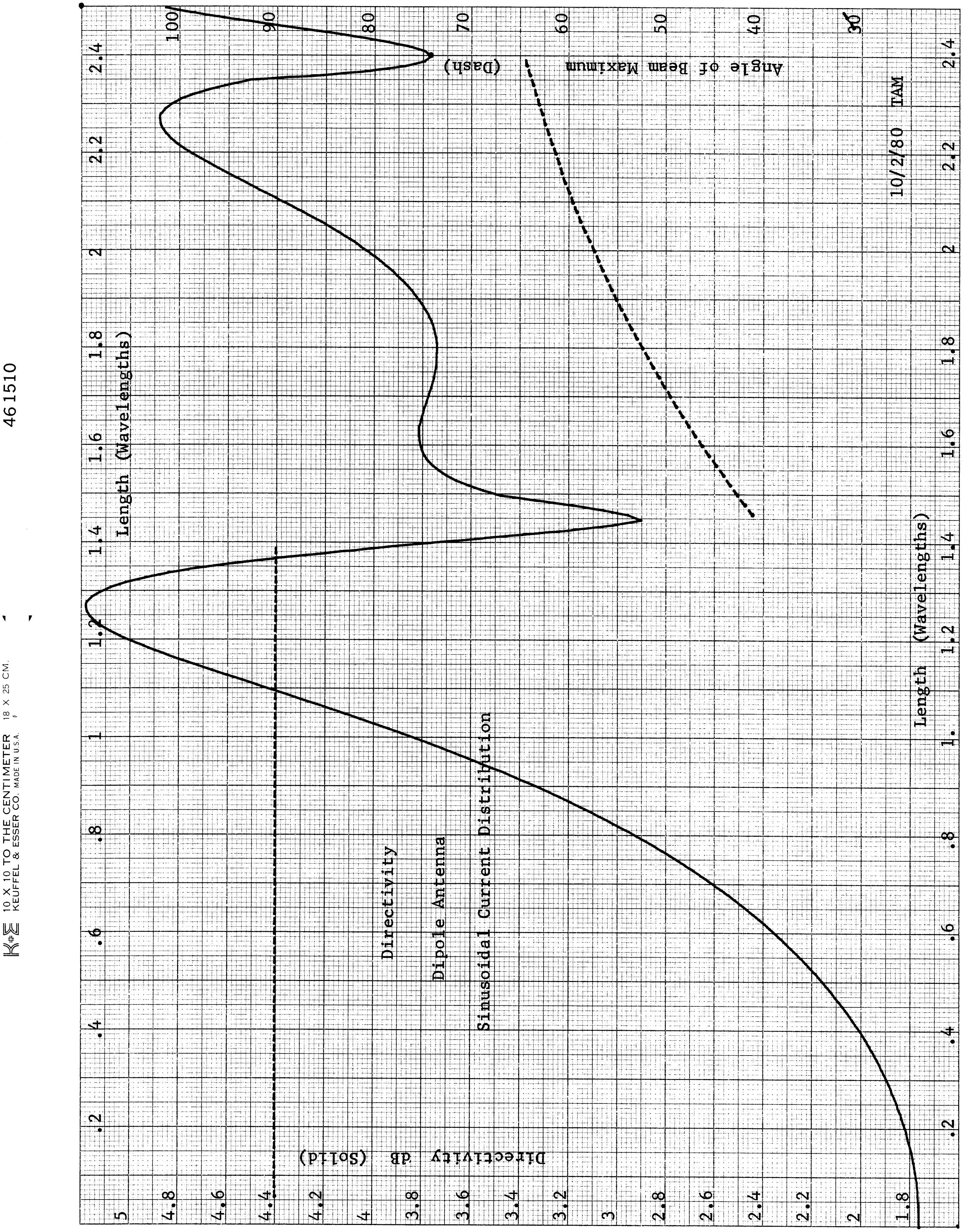
$$\text{Directivity} = \frac{\eta \left[1 - \cos\left(\beta \frac{L}{2}\right)\right]^2}{\pi R_r} \qquad L \leq \lambda$$

On page 134 is a curve of the gain (directivity) of a symmetric feed dipole with a sinusoidal current distribution. The dipole has such low loss if it is matched to the transmission line, that directivity is taken for gain in many cases. We can see that the half-wave dipole has a gain of 2.15 dB and the incremental dipole a gain of 1.76 dB.

The peak of the beam is plotted as a dashed curve on page 134. Note that the peak of the beam location is discontinuous. This can be seen by looking at the two polar plots on page 135. The solid curve is for a dipole 1.4 wavelengths long. The dashed curve is for a dipole 1.45 wavelengths.

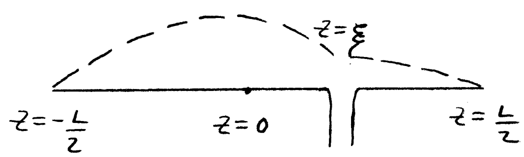


Length (Wavelengths)
.1 .2 .3 .4 .5 .6 .7 .8 .9 1 1.1 1.2 1.4
Resistance (ohms) (Solid)
450
400
350
300
250
200
150
100
50
Reactance (Dashed)
200
150
100
50
0
-50
-100
-150
-200
Reactance
Diameter
0.08λ
0.04λ
0.02λ
Resistance
Symmetric Dipole
Input Impedance
10/3/80
TAM
Length (Wavelengths)
.3 .4 .5 .6 .7 .8 .9 1 1.2 1.4
46 1320
10 X 10 TO ½ INCH 7 X 10 INCHES
KEUFFEL & ESSER CO. MADE IN U.S.A.


Directivity
Dipole Antenna
Sinusoidal Current Distribution
Length (Wavelengths)
Directivity dB (Solid)
Angle of Beam Maximum (Dash)
10/2/80 TAM
10 X 10 TO THE CENTIMETER 18 X 25 CM.
KEUFFEL & ESSER CO. MADE IN U.S.A.
46 1510

OFFSET FEED

All of the dipoles we have looked at so far have had symmetrical feeds. We will now look at the case where the antenna is fed elsewhere. The same coordinate system will be used for this as the symmetric feed. The center of the coordinates is still at the center of the length of wire. See figure. For this case we will still assume sinusoidal current distributions. The currents must be zero at the ends of the wires. The proper functions are



$$I(Z) = I_1 \text{ Sin } \beta\left(\frac{L}{2} - Z\right) \qquad Z \geq \xi$$

$$I(Z) = I_2 \text{ Sin } \beta\left(\frac{L}{2} + Z\right) \qquad Z \leq \xi$$

These functions vanish at $Z = \pm L/2$. Assuming we have a balanced line feeding the antenna, the currents entering the antenna are equal and opposite. This gives us continuity of current at $Z = \xi$.

$$I_1 \text{ Sin } \beta\left(\frac{L}{2} - \xi\right) = I_2 \text{ Sin } \beta\left(\frac{L}{2} + \xi\right)$$

This will be satisfied if

$$I_1 = A \text{ Sin } \beta\left(\frac{L}{2} + \xi\right) \text{ and } I_2 = A \text{ Sin } \beta\left(\frac{L}{2} - \xi\right)$$

Where A is a constant. The current in the antenna is given by

$$I(z) = A \sin\beta\left(\frac{L}{2} + \xi\right) \sin\beta\left(\frac{L}{2} - z\right) \qquad z \geq \xi$$

$$= A \sin\beta\left(\frac{L}{2} - \xi\right) \sin\beta\left(\frac{L}{2} + z\right) \qquad z \leq \xi$$

If we look at the case where the length, L, is one wavelength, we find two equally likely current distributions. When the antenna is fed symmetrically, then the currents are symmetric as in figure A below. But if the antenna is offset fed, then the current distribution in B

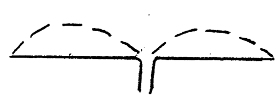


Figure A

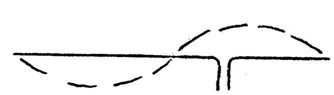
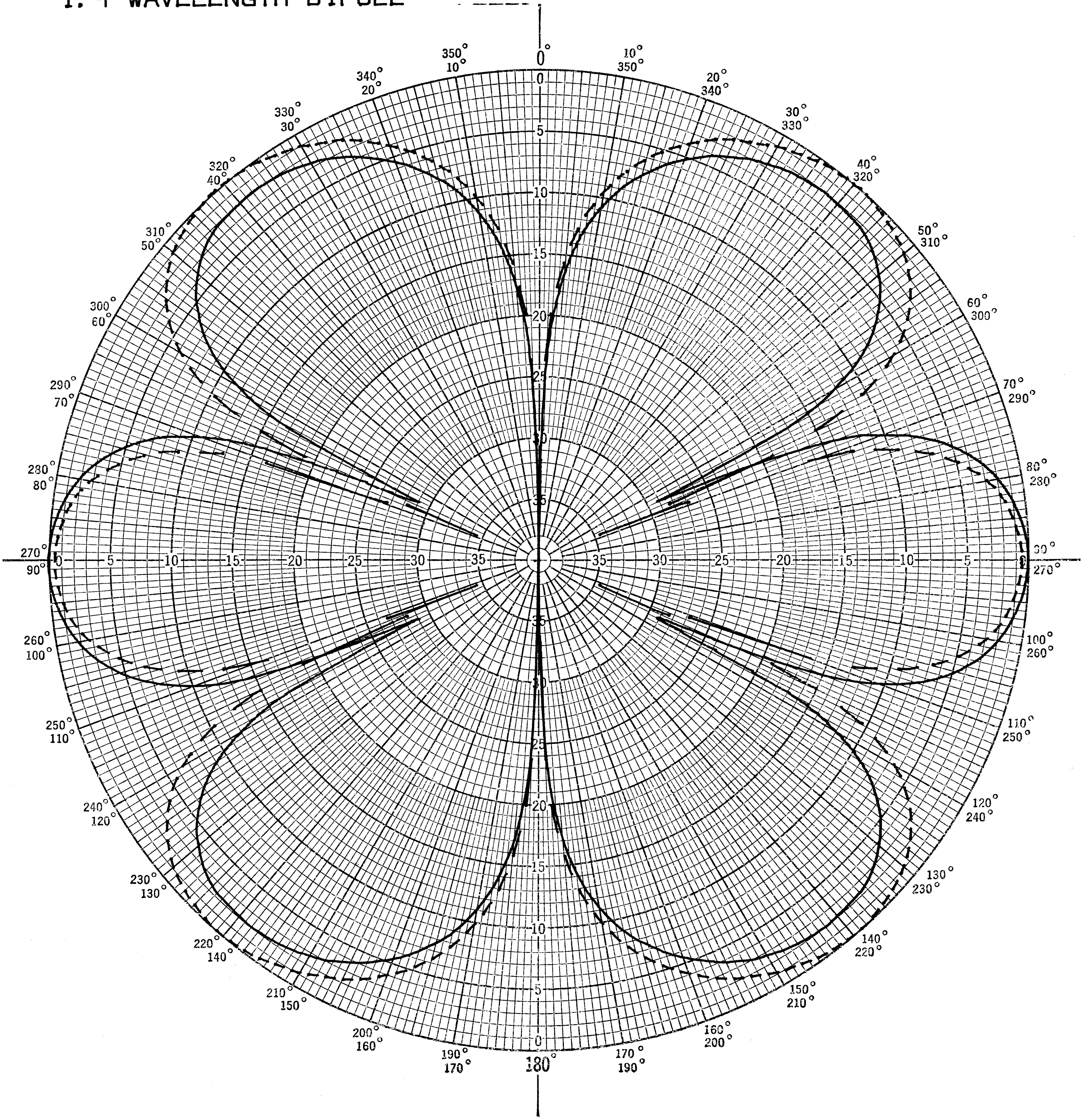


Figure B

is obtained. We can see the results of the two distributions in the plots on page 137. The first case is symmetrically fed and the second asymmetric. The asymmetrically fed has a null at theta equals 90 degrees as would be expected when we consider a line parallel to the Z axis and look at the sum of the partial fields from each part of the antenna current. We must keep in mind that all these current distributions are approximations and real antennas will only approximately follow these plots. Also there is no value where the two distributions will switch.

1. 4 WAVELENGTH DIPOLE



Polar Chart No. 127D
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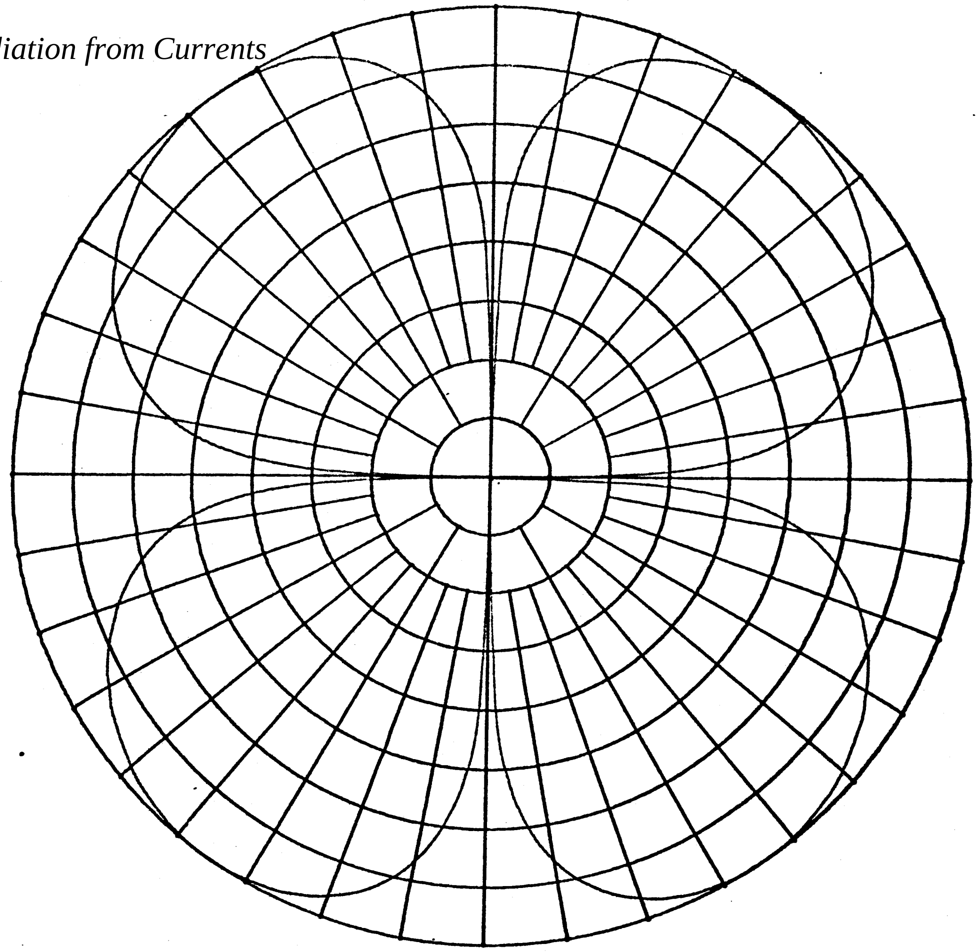
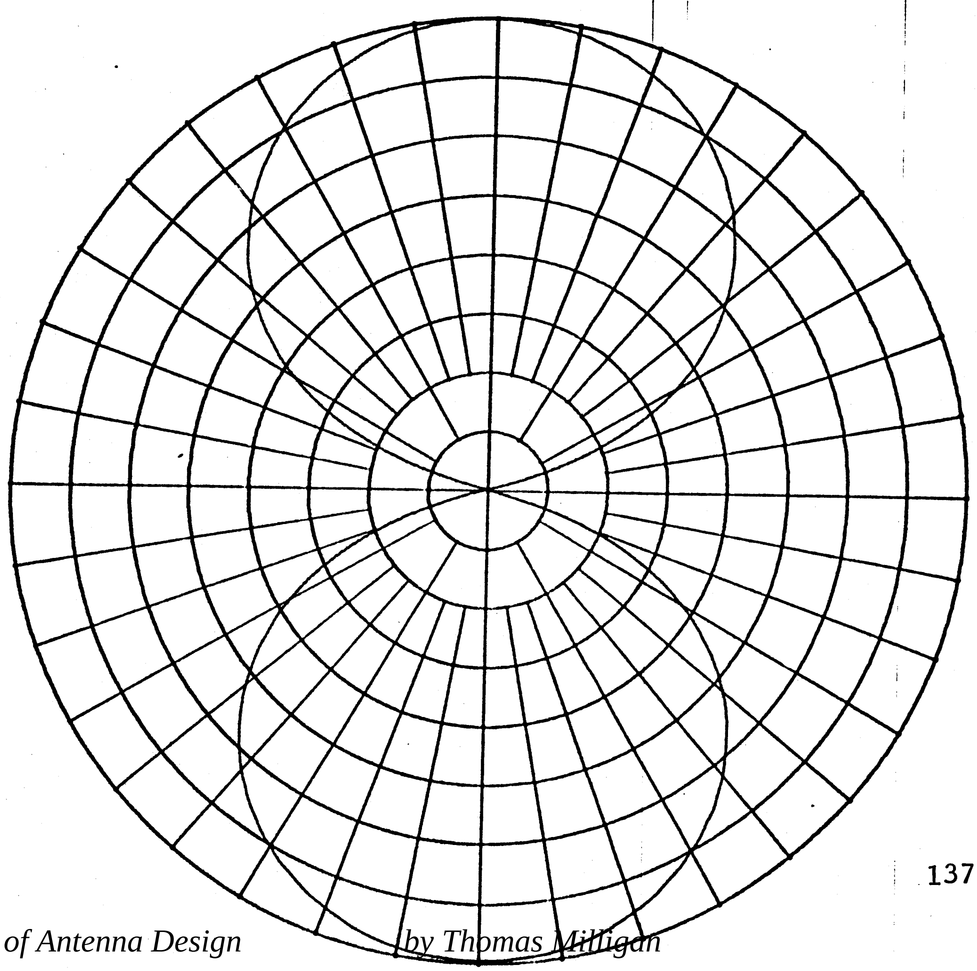


FULL WAVELENGTH DIPOLE OFFSET FEED



FULL WAVELENGTH DIPOLE, SYMMETRIC FEED

The input impedance in figure A on page 136 is quite high, but it is quite reasonable in figure B of the one wavelength long antenna. In figure B the input is at a high current, low voltage point (standing waves) and will have an input resistance close to the radiation resistance given on page 131, 200 ohms. The feed in figure A is at a low current, high voltage point which is a high impedance point. In general low current points are high input resistance points and high current points are low input resistance points.

BALANCED FEED

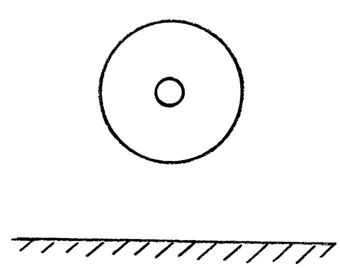
I have stated that the dipole has been fed with a balanced feed. We must investigate what is meant by this and what an unbalanced feed is and its effect. We have not considered the effect of a ground plane on an antenna but we will now consider the effect on the transmission line feeding the dipole antenna.

We are used to thinking that one of the transmission line conductors is the ground line but this leads us to incorrect results. When the antenna is over a ground plane, which almost all dipoles are, then we can consider the transmission line feeding it to be a three wire line. In general, the two conductors leading to the antenna terminals do not necessarily have equal currents. There are balanced and unbalanced transmission lines with respect to ground. Between balanced and unbalanced lines there should be special structures called baluns which properly connect the two different transmission lines.

An unbalanced line has two conductors which have different potentials with respect to ground. For example, one of the conductors may be ground. In constrast, a balanced line will have equal and opposite potentials with respect to ground. The unbalanced line conductors have different capacitances to ground and the balanced line equal capacitances. We extend this to say that the two conductors of a balanced line carry equal and opposite currents. It is important in antenna theory to take this definition because currents are induced in the conductors from external electromagnetic waves.

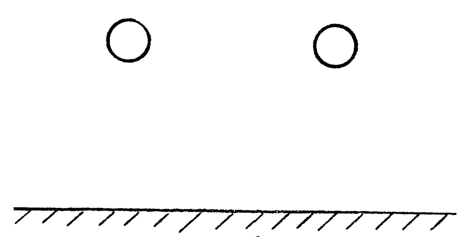
The two examples of the unbalanced and balanced lines are the coax line and the two wire line.

Unbalanced Line



Ground

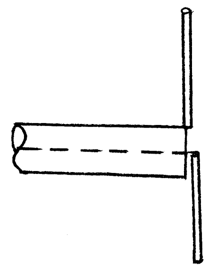
Balanced Line



Ground

The inner conductor of the coaxial line has no direct capacitance to ground whereas the outer shield has direct capacitance to ground. It is unbalanced with respect to ground. The two wire line over ground has equal capacitances to ground from each of the two lines. It is a balanced line.

Let us consider a dipole in reception which has a coaxial feed line. When we place this in a plane wave, currents are induced on the arms of the



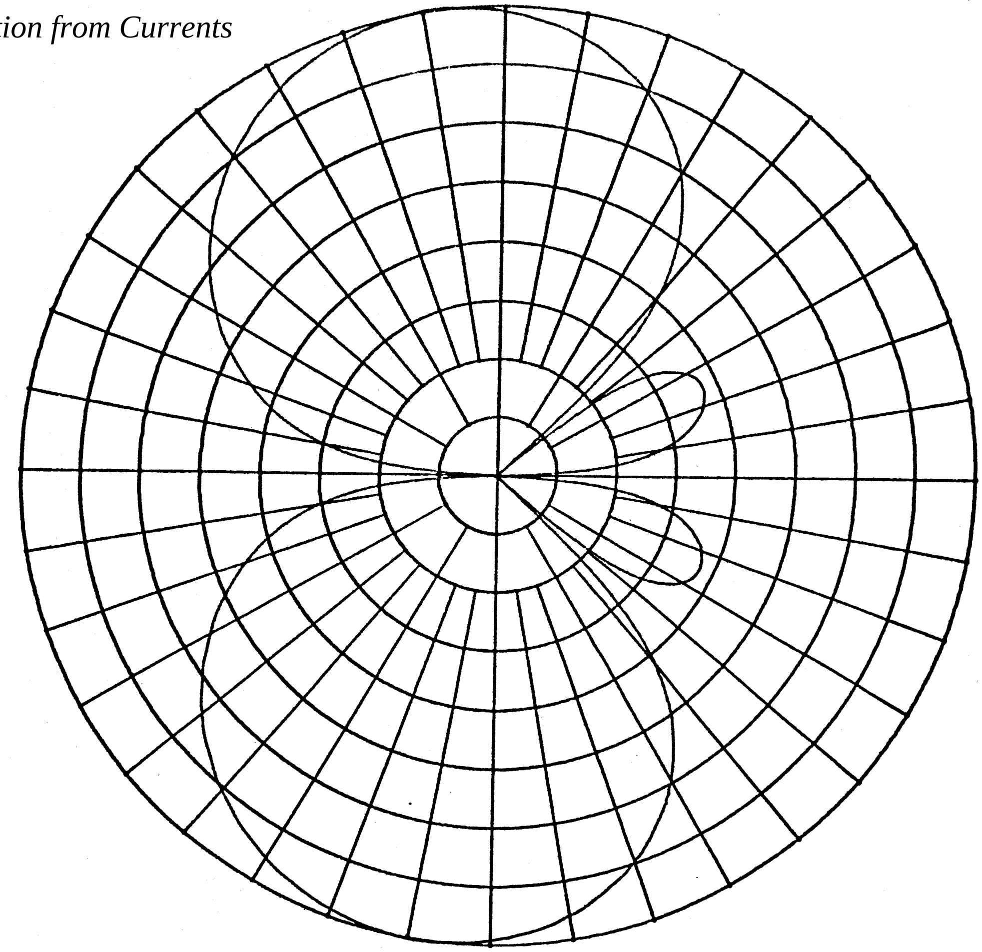
dipole and on the outer shield of the coax line. The inner conductor is shielded from the passing wave and has no currents induced on it. The currents induced on the outer shield and the currents induced on the dipole arms add giving unequal currents on the two conductors of the feed line. The antenna feed is unbalanced. If we now connect the antenna with a two wire transmission line then the currents induced on the two conductors by the passing plane waves will be equal and in the same direction. When connected by a coax line the dipole is able to receive energy from the passing plane wave when it originates from the direction $\theta = 0$ which the dipole itself cannot. The two wire line has equal currents induced on each line which do not enter the receiver connected across the lines.

Now let us turn the problem around and consider the dipole antenna transmitting. When fed with a coax, the current flows out of the inner conductor and on to one of the arms of the dipole. What happens to the current on the outer conductor? Most of it flows out on to the other arm of the dipole, but some of it can also flow down the outside of the coax. The outer conductor forms a transmission line between it and ground also. At high frequencies the current only flows on the surface of a conductor. This current flowing on the outside of the coax has two effects. First it radiates because there is some distance between it and the ground return path which is in the ground plane. Secondly it unbalances the currents on the dipole. Unbalanced currents on the dipole usually has the biggest effect. On page 140 are polar plots of a half-wave and full-wave dipole antennas fed with unbalanced currents on the two arms. The peak of the beam is no longer at $\theta = 90^{\circ}$. When the peak of the beam is not as it should be from the theory, we call this a squinted beam. The squinted beam is caused by balance problems on the antenna feed lines.

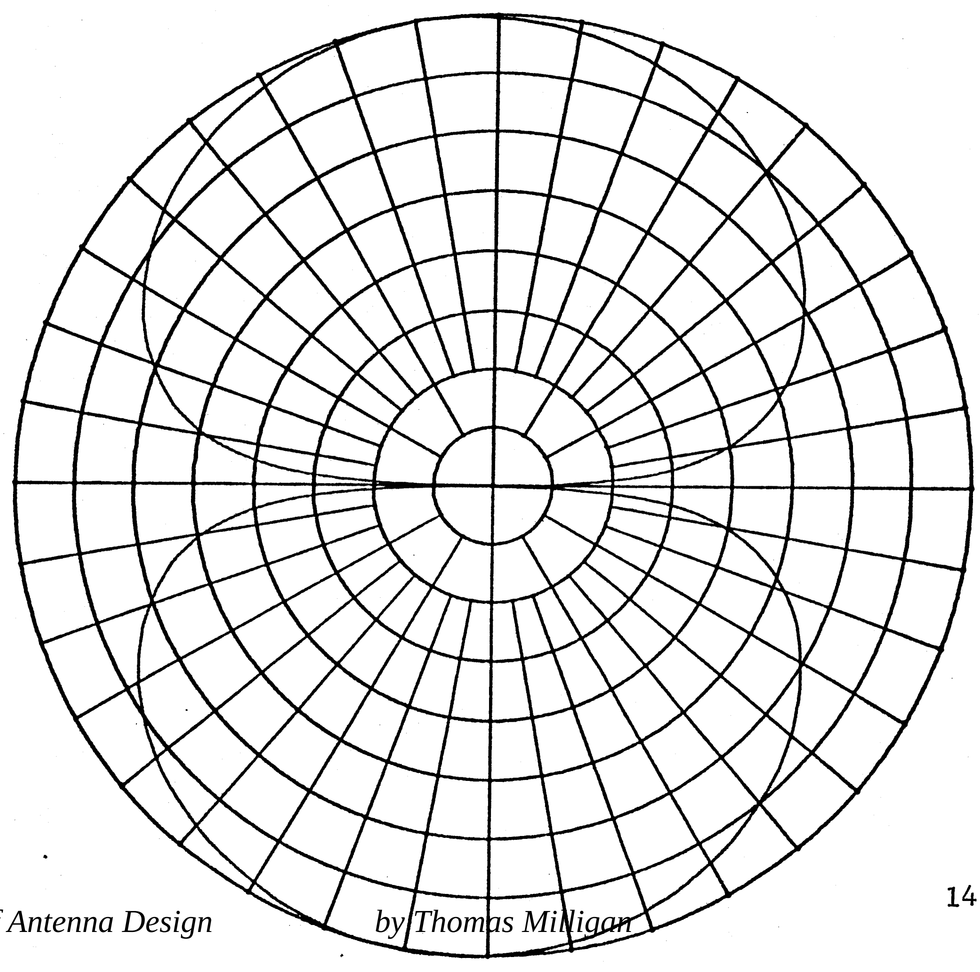
BALUNS

A balun is a circuit device which properly connects a balanced transmission line to an unbalanced line. Most baluns can be understood by simple arguments about the impedances to the balanced and unbalanced modes on the three wire transmission line. It will be helpful to discuss the fundamental modes on a three wire line.

FULL WAVELENGTH DIPOLE WITH UNBALANCED FEED

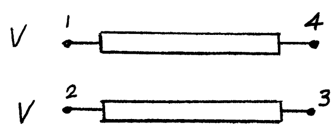


HALF WAVELENGTH DIPOLE WITH UNBALANCED FEED

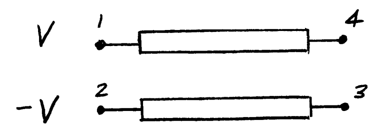


BALANCED - UNBALANCED/ ODD - EVEN MODES

A group of wire lines have certain fundamental modes by which the TEM waves traveling on it can be described. These fundamental modes form a basis set for all possible modes. A two wire line has only one fundamental mode. A three wire transmission line has two fundamental modes. These modes are traditionally called the even and odd modes. The general N conductor transmission line has N - 1 fundamental modes. These even and odd modes can be related to the unbalanced and balanced modes. We will consider the following two diagrams of the three wire line where the ground plane (or conductor) is implied.

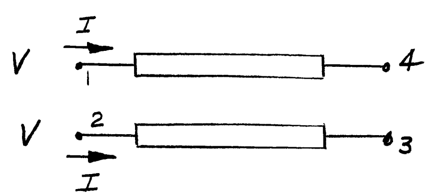


Even Mode

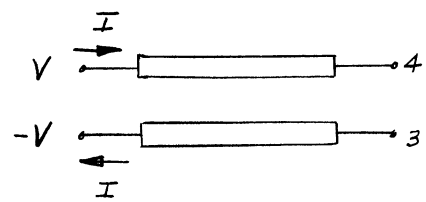


Odd Mode

The voltages are with respect to the ground plane. When the ends of the two transmission lines are loaded equally to ground on ports 3 and 4, then the lines will have equal currents on the lines. The currents are given by:



Unbalanced Mode
(Even mode)



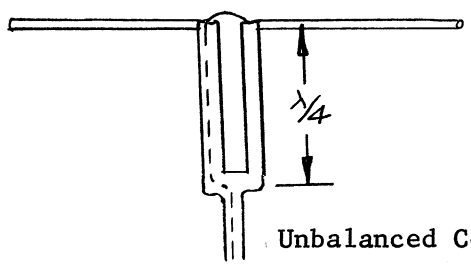
Balanced Mode
(Odd Mode)

The even mode corresponds to the unbalanced mode and the balanced mode to the odd mode. A current into the ports 1 and 2 can be described as a combination of the balanced and unbalanced modes. Similiarly the voltages can be given as components of the even and odd modes. The proper currents feeding a balanced transmission line are of course the balanced mode or the odd voltage mode. Either mode can be excited on the three wire transmission line or any combination of the two modes.

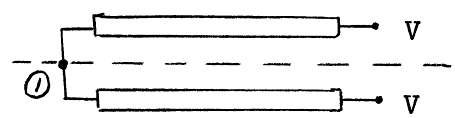
The function of baluns is to eliminate the excitation of the unbalanced mode on the three wire transmission line. It can be equally said that the balun prevents the unbalanced mode currents from being coupled into the receiver. Hopefully this will become clearer as various baluns are discussed.

A balun is needed because most receivers have unbalanced mode inputs. The second reason is that it is convenient to route signals through coaxial transmission lines. The two wire balanced line requires space between the different sets of transmission lines to prevent coupling of different signals. Below we will discuss most of the common balun structures.

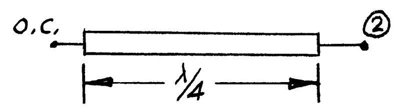
Folded Balun - The figure below is a balun which allows a coax feeder to be connected to a dipole antenna.



This structure can be analyzed using balance (odd) and unbalanced (even) mode analysis. The outer shield of the coax and the extra line are two lines in a three wire transmission line for the unbalanced mode.

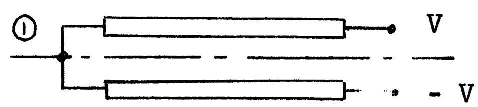


In the even mode (unbalanced) there will be a magnetic wall between the conductors. A magnetic wall is a boundary where the magnetic field vanishes which implies an open circuit at ①. We can now analyze the unbalanced mode as a single line.

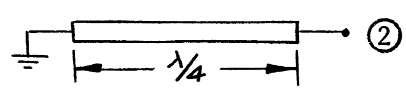


The impedance at ② will be that of a quarter wavelength transmission line to an open circuit. The input impedance to the unbalanced mode is a short circuit at the dipole arms. The power transferred to the coax from the unbalanced mode is zero since the dipole arm and the short circuit are in shunt across the coax. Any unbalanced mode currents induced on the dipole or the coax outer shield will be shorted out at the input to the coax.

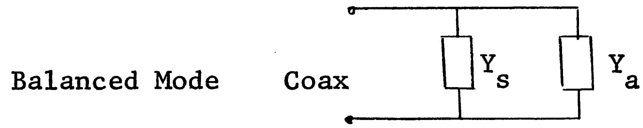
In the odd (balanced) mode there will be an electric wall between the conductors. An electric wall is a boundary where the electric fields



vanish. This implies a short circuit at ① in the above diagram. We can now analyze the balanced mode as a single line. The impedance at ② will



be an open circuit. A quarter wave transmission line transforms a short circuit to an open circuit. To the balanced mode the circuit is two admittances in parallel at the output of the coax.

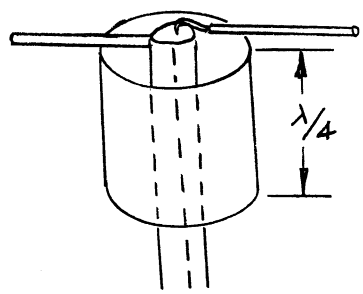


$Y_s = -j\ Y_o \text{ Cot } K\ell$

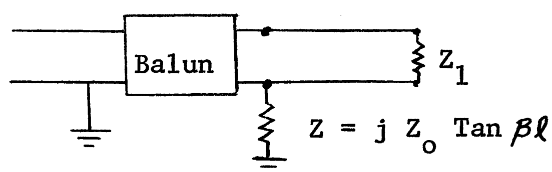
Quarterwave line with a short Termination

If the even and mode impedances are known, then the power transfer to both modes can be found for any frequency to find the frequency response. This balun is inherently narrow band but has a wider bandwidth than a half-wave dipole.

Sleeve or Bazooka Balun - The figure below is the bazooka balun. The balun prevents currents on the outside of the coax feeder. The quarter-wave

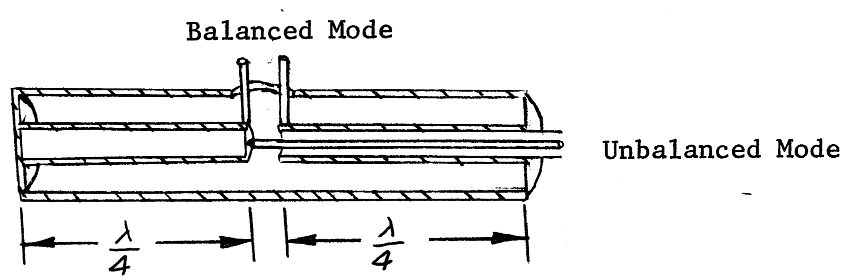


shorted outer coax sleeve will present an open circuit to the unbalanced currents on the outside of the coax. In a sense the second coax sleeve or skirt has shielded the inner coax. It is reported that this balun is very narrow band. Sometimes a second skirt is placed below the first one and facing the other direction to open circuit currents flowing up the coax line outer shield from below. The equivalent circuit of the sleeve balun is given below.

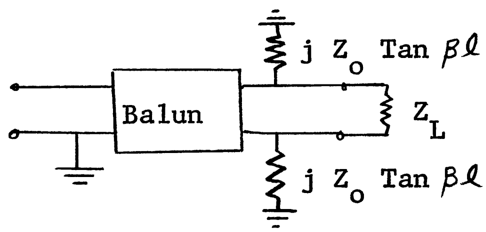


The outer shield of the coax path to ground is through the quarter-wave stub. As the frequency is changed the impedance to ground of the outer shield reduces and the currents become unbalanced.

Type II Sleeve Balun - The bandwidth of the sleeve balun can be increased by adding a second stub to the inner conductor of the coax. The diagram below is the Type II Sleeve Baluns in cut away.

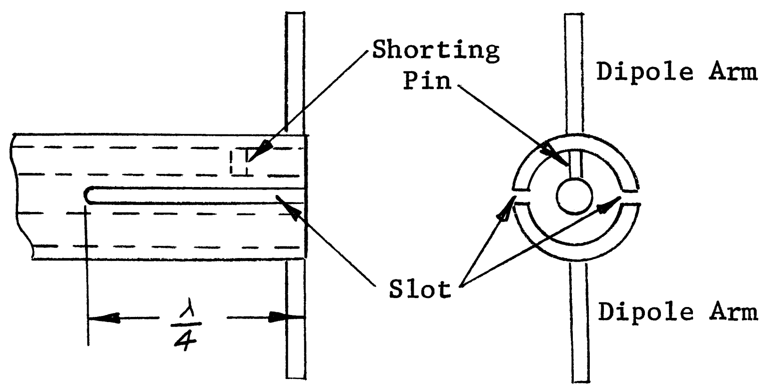


The outer shield of the coax is shielded from unbalanced fields. The diagram below is the circuit diagram of the balun.



Because the extra short circuited stub has the same characteristic as the first stub, the two output lines have the same impedance to ground and the balanced lines track with frequency changes. The balun is balanced at all frequencies except there is a shunt stub across the input line which will limit the frequency bandwidth of the balun when feeding an antenna.

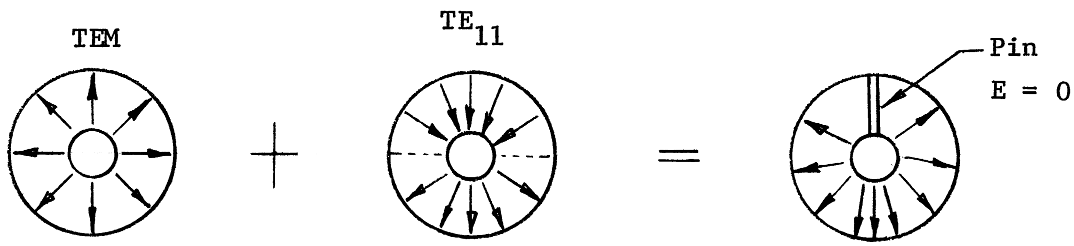
Split Coax Balun - This balun allows both arms of the dipole to be connected to the outer shield of the coax. It is a good structure because symmetry can be maintained and it is quite rigid which helps overcome vibration problems. The configuration is given below.



This structure works by supporting two coax modes simultaneously on the section with the two slots. The two modes make it the same as a three wire line. The two modes are

The TE_{11} mode is a coax waveguide mode which is the balance (odd) mode and it excites the dipole arms. The TEM mode is the unbalanced mode to the dipole arms. This structure is similiar electrically to the folded balun. In the even (unbalanced) mode there is a virtual open circuit at the point where the slots end. The open circuit is transformed to a short circuit at the dipole arms and decouples this mode from the arms of the dipole.

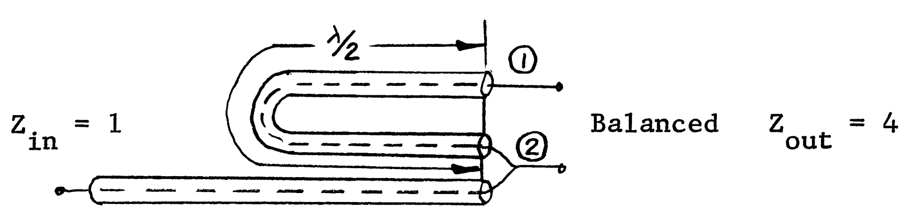
The TE_{11} mode is excited by the pin across the two conductors of the coax. The following digram explains the excitation.



The pin across the coax causes the electric field to be zero at that clock angle. The fields which satisfy these boundary conditions are the superposition of TEM and TE_{11} modes. Usually the coax will not support the TE_{11} mode unless the coax has two slots. It does not matter where the pin is located in the slot region. Usually the pin is at the end because it is easiest to place it there. Again this balun is similiar electrically to the folded balun in the balanced mode. At the point where the slots end the TE_{11} mode is short circuited. The arms of the dipole are a quarter wavelength away from the short. The slotted coax presents an open circuit and it does not effect the power transfer to the balanced mode. This is a shunt stub across the dipole arms. The bandwidth of this structure is comparable to the dipole.

The split coax balun and the folded balun are similiar electrically but the symmetry of the split tube balun is an advantage. The shorting pin is used only to excite the TE_{11} mode which feeds the dipole arms. The currents are exactly 180^o out of phase. The folded balun uses a short piece of wire to connect to the second dipole. The length of wire is an inductor which introduces phase shift to the center fed dipole element and squints the pattern. As the frequency increases the split coax balun will be the better balun. The same problem occurs with the "infinite" balun of the log periodic antenna.

Half-wave Balun - This balun not only connects an unbalanced line to a balanced line but it also transforms the impedance by a factor of 4. This balun works by cancellation of the unbalanced mode currents at the input to the coax. Below is the diagram of the balun.

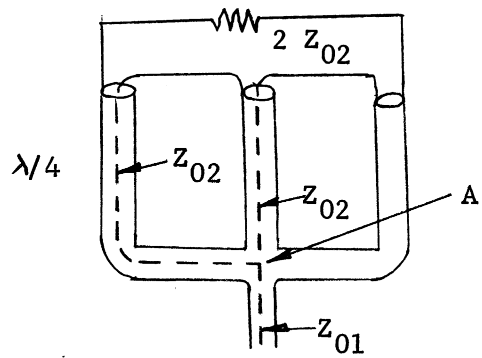


Consider the load across a balanced mode transmission line. Since it is in the odd mode, there will be a virtual short half-way through the load. To each line the load impedance to ground will be half the total impedance between the lines. On the ports 1 and 2 above the impedance will be $2\ Z_o$ in the balanced mode. The load on 1 will be transformed through the half wave transmission line to the same impedance at position 2. These two load impedances are in shunt at point 2. The two loads are each $2\ Z_o$; combined in shunt the input impedance is Z_o, the characteristic impedance of the feeder.

The signal routine establishes the output mode. In the balanced mode the voltage at 2 is V_o. When the voltage wave is transmitted through the half wavelength line, the sign of the voltage changes. The voltage at point 1 is 180^o out of phase with the signal from point 2 and the line is in the balanced mode. Now in the unbalanced mode the signal from 1 arrives with an angle of 180^o at port 2. This signal is cancelled by the signal directly from the dipole arm on 2. There is no transfer to the unbalanced mode from signal cancellation.

This balun is a popular balun for low frequency antennas because it is easily made from coax cable. The half-wavelength cable can be rolled up into a small space. Also many antennas such as the folded dipole have impedances around 300 ohms. With a 300 ohm antenna the balun is made from 75 ohm coax cable such as RG 59 and transforms the antenna impedance to 75 ohms.

Candelabra balun - This is another balun which is also a four to one impedance transformer from input to output. The characteristic impedance of



the unbalanced input is Z_{01}. At point A the coax is divided into two shunt connected coax lines. In order that the lines be matched, the characteristic impedance in both these lines should be twice Z_{01} or Z_{02}. At the end of the quarter-wave section in air, the impedance across the coax lines is Z_{02}. These two lines are connected in series across the load. To have a matched system, the load must be twice Z_{02}. In the balanced mode the transmission line between the outer shields or the outer shield and the dummy line are shunt quarter-wave shorted stubs across the lines. Of course, to the balanced mode these stubs have no effect because they are open circuits.

The unbalanced mode currents are shorted out at the input to the coax lines in the same manner as the folded balun. There is a magnetic wall between the outer shields of the coax lines. Open circuits are half way between the coax connections at the base. The open circuits are transformed through the quarter wavelength sections of two wire line to short circuits at the input to the coax lines to the unbalanced mode currents.

The output impedance is four times the input impedance and in the balanced mode. More lines can be stacked in series and higher input to output impedances can be designed. Higher impedance ratios are impractical.

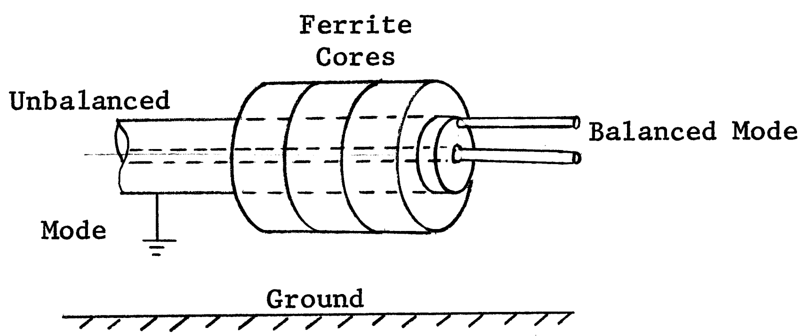
FERRITE CORE BALUNS

Ferrite material can be used to increase the impedance to the unbalanced mode and eliminate it. Ferrite has a high relative permeability and also has increasing loss as the frequency increases. It will raise the inductance per unit length if it is placed in the transmission line between the conductors. From transmission lines we can recall that the characteristic impedance of a transmission line is

$$Z_o = \sqrt{\frac{L}{C}}$$

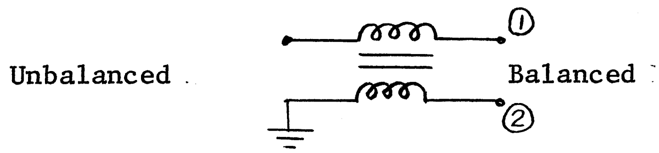
If the inductance per unit length is increased by using ferrite, the characteristic impedance increases.

Ferrite Bazooka Balun - There is a ferrite core equivalent to the bazooka or sleeve balun. Ferrite cores are placed on the outside of a coax line.



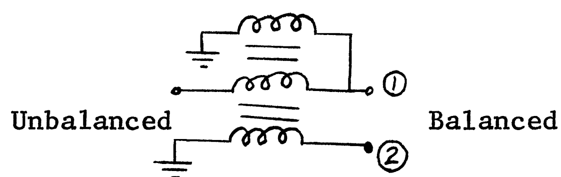
The ferrite cores will increase the impedance to ground for currents on the outside of the coax line. Like the bazooka balun they present an open circuit to these currents. The ferrite cores inhibit the unbalanced currents between ground and the outer shield of the coax. The advantage of the ferrite core balun is that it is not frequency sensitive; it can work over many decades. The impedance between the outer shield and ground is named Z_p. As the impedance due to the ferrite material starts decreasing, the length of transmission line between the outer shield and ground will approach $\lambda/4$ and increase the bandwidth. At even higher frequencies, it will start approaching $\lambda/2$ and the balun will fail. Any ferrite core balun is a compromise design between the low frequency response which is controlled by the amount of ferrite and the high frequency response which is controlled by the length of transmission line.

One to One Ferrite Balun - This ferrite balun is similiar electrically to the bazooka ferrite balun. In this case bifiliar wire windings are used for the transmission line. Wire can be purchased in bifiar form which closely approximates 50 to 100 ohm characteristic impedances. These wires range from #36 to #38 wire size. The wire is wound around the ferrite core.



This balun has the same input and output impedances. In this case the windings present a high impedance to ground from point 2. It is like the bazooka balun ground load.

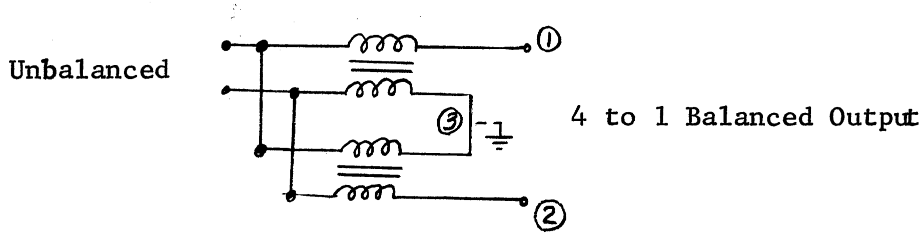
A balun like this can work satisfactorily from 100 KHz to 1 GHz and above or four decades. The balun can be thought of as follows: when there are balanced currents in the windings, the equal and opposite currents will produce fields in the ferrite material which will cancel each other. In the unbalanced mode the fields in the ferrite core material will add and each winding sees the ferrite core. Both windings will have high inductances and resistance to ground. If a more detailed analysis of the balun is performed, we would find that the amplitude at port 2 will be less than that at port 1. It is due to the losses introduced in the ground lead by the ferrite core. This can be corrected by adding an extra winding to ground. It is the same as the Type II bazooka balun.



This balances the balun outputs by adding loss to port 1 output. There is no increase in the bandwidth with the extra winding, just better balance.

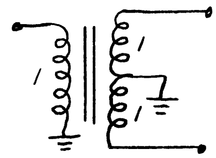
FOUR TO ONE FERRITE CORE BALUNS

<u>Candelabra ferrite balun</u> - There is a ferrite core equivalent to the candelabra balun. It too has a four to one impedance ratio.



This is a parallel to series transformer; the two outputs 1 and 2 are matched to four times the input impedance. Like the coax version the characteristic impedance between the wires in the cores should be twice the source impedance. At point 3 there is a virtual short; in some versions this point is shorted to ground. It sometimes helps the balance.

Transformer Balun - This balun has no transmission line equivalent, but it is merely a transformer. The following is the circuit diagram.



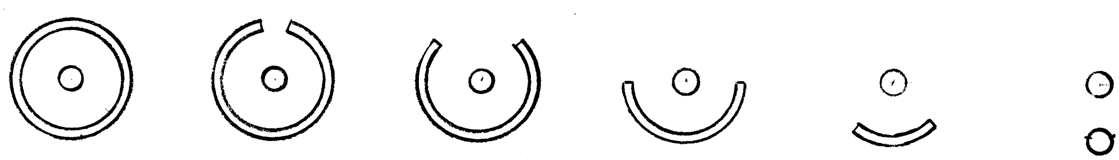
We can see that the impedance has been increased by four from the transformer relation since the windings on the output have twice as many turns. When we look at the output, the output ports see the same impedance to ground. This balun is not helped at the high frequency end by transmission line balun action. It has limited high frequency response. But it is a good balun at low frequencies.

One of the problems with ferrite core baluns is the limited power handling capability. Most of them are made from #36 or #38 wires. This limits their use to receive only powers.

SPLIT TAPERED COAX BALUN

This balun is unlike all the others. It can only be understood from the possible currents induced on it by passing fields or from the field generated on the arms of the dipole. The other way to look at it is from a structures point of view. It starts with a line which is unbalanced to ground and ends in a balanced two wire line. The balun and its construction is described in Proc. IRE, February 1960, by J. W. Duncan and V. P. Minerva.

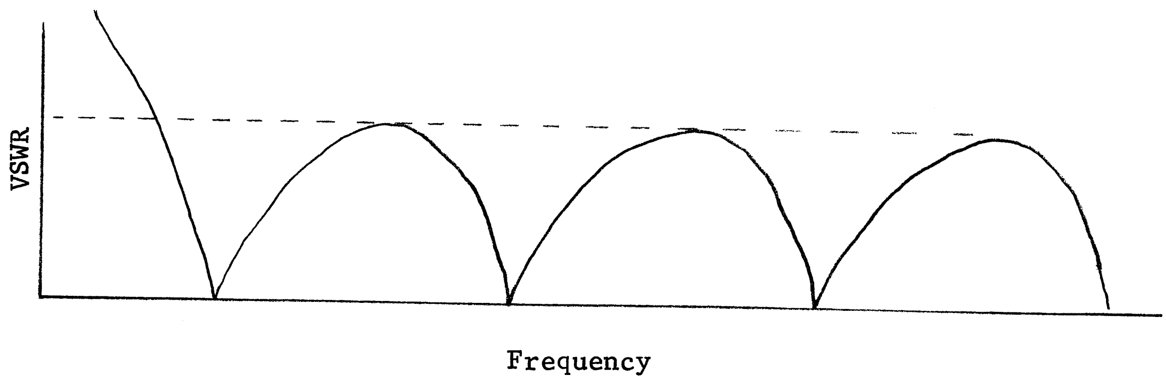
The balun starts with a coax line which is inherently unbalanced. The coax has a slot cut in it. As we move down the line, the slot or split is opened up more and more exposing the inner conductor.



Cross Sections Along Balun Length

Above is a series of cross sections down the coax balun as the split grows in size. The balanced line has a higher impedance than the coax and as the balun progresses down the opening coax, the impedance is transformed from the lower coax impedance to the higher two wire line impedance.

Duncan and Minerva picked a tapered transformer which would transform the impedance from one end to another and be broadband. The choice they picked was the Dolph-Tchebycheff transformer. This transformer has uniform mismatch ripples as the frequency increases and potentially has no upper frequency bound. It starts after some minimum frequency. The VSWR characteristic is given on the following graph.



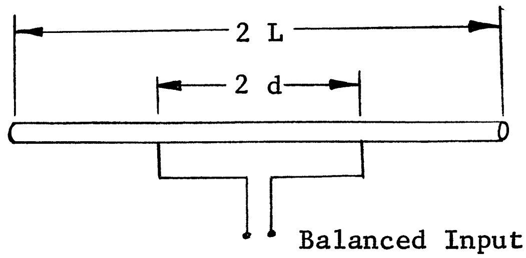
The returns loss of the transformer is a measure of the level of the unbalanced mode because this is a current traveling in the other direction on the coax. This transformer is not only possible transformer. I have built these with exponential tapers as well. The exponential taper is longer and the ripples in the return loss decreases with increasing frequency. The Dolph-Tchebycheff transmission line transformer has finite impedance steps at the beginning and end of the transformer which the exponential taper does not have. The exponential taper or a smooth transition taper seems to have a higher frequency response, although this has been hard to demonstrate. The smooth taper will have better high frequency response because the ripple will die down with increasing frequency.

This balun can also be built using microstrip construction. In this case the ground plane is tapered down until both the upper and lower conductors have the same size. A balun can be built which will work from 100 MHz to at least 18 GHz using this technique. I have used this in mixer circuits but there are few antennas which can cover this wide a bandwidth. The only trouble with this balun is that the impedance is increased in the balun.

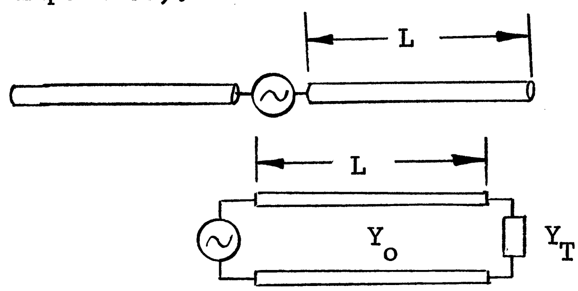
This does not exhaust the possible baluns which have been published. The so called "infinite" balun used with log periodic antennas will be described when they are covered. When a broad beam antenna is designed, sometimes a little squint in the beam is not too important and an acceptable design can be achieved without using a balun. There are a few baluns which have been designed and not published since certain companies have felt it has given them a competitive edge and they are difficult to copy.

SHUNT FEEDING

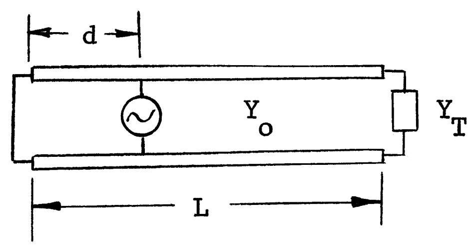
Shunt feeding is a method of increasing the input impedance of a halfwave dipole antenna. This configuration is given below.



The dipole can be thought of as a transmission line with a terminating load (the radiation impedance).



When the antenna is fed at some point d, the equivalent circuit is

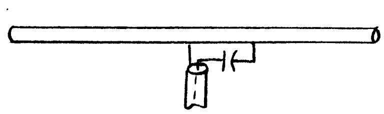


The input admittance is given by

$$Y_{in} = -j\,Y_o \operatorname{Cot}\beta d + Y_o \left[\frac{Y_T + j\,Y_o \operatorname{Tan}\beta(L-d)}{Y_o + j\,Y_T \operatorname{Tan}\beta(L-d)}\right]$$

The first term is the shunt shorted stub. The second is the termination admittance at the end of the section of transmission line (the dipole) of length L - d. The input admittance is inductive due to the shunt short circuited stub. This inductance can be resonated out by inserting a series capacitor in the feed line. The resistive portion of the input impedance of a half-wave dipole is increased because it is fed at a lower current point. This feeding is sometimes called T match or delta match.

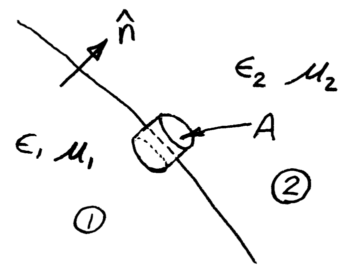
The antenna can be fed in an unbalanced mode as in the following diagram.



This is called a gamma match. Both of these feed systems have the advantages of increasing the input impedance, of making the dipole a single solid line, and in the case of a monopole of grounding the antenna which protects large towers from lightning. Even in smaller antennas it prevents static charge build up. It is usually easier to tune the antenna by hand than to calculate the position of the tap.

BOUNDARY CONDITIONS

Suppose there are two different mediums separated by some boundary. We need to know what happens to the fields as they cross the boundary.



Consider a small pill box which sits astride the boundary. From one of the Maxwell's equations we can take the surface integral of the magnetic flux density.

$$\oint_S \bar{B} \cdot d\bar{S} = 0$$

The surface is over the whole pill box. The pill box can be made as thin as wanted and still retain the areas on the ends in the two mediums. The integral then reduces to

$$(B_{n2} - B_{n1})\, A = 0$$

where B_{n2} and B_{n1} are the normal components of the magnetic flux density on the ends of the pill box. Since the area, A, is arbitrary, this must hold for any area. The conclusion is that the normal component of the magnetic flux density is continuous across the boundary.

$$B_{n1} = B_{n2}$$

Vectorially this expression is

$$\hat{n} \cdot (\bar{B}_2 - \bar{B}_1) = 0$$

where $\hat{n}$ is the normal unit vector to the plane defined at the point on the boundary and $\bar{B}_2$ and $\bar{B}_1$ are the magnetic flux densities in medium 2 and 1. $\hat{n}$ points into medium 2.

The magnetic field is related by the permeability to the magnetic flux density. The boundary condition for the normal component of the magnetic field.

$$\mu_1 H_{n1} = \mu_2 H_{n2}$$

If the medium 1 is a conductor, then no fields can exist inside the conductor and the normal magnetic field (or flux density) will be zero.

We can use the same pill box to find a relationship between the normal electric flux densities. From Gauss's law the surface integral of the electric flux density over a closed volume is equal to the charge enclosed by the surface.

$$\oint_S \bar{D} \cdot d\bar{S} = Q$$

Since the height of the pill box is shrunk to an infinitesimal height, the only charge enclosed would be a surface charge density, σ_s, times the area.

$$(D_{n2} - D_{n1})\, A = \sigma_s\, A$$

$$D_{n2} - D_{n1} = \sigma_s$$

The vectorial expression is

$$\hat{n} \cdot (\bar{D}_2 - \bar{D}_1) = \sigma_s$$

Of course, if there is no trapped charge on the surface, then the normal component of the electric flux density is continuous across the boundary.

The electric field is related to the electric flux density by the permittivity of the two mediums.

$$\hat{n} \cdot (\epsilon_2 \bar{E}_2 - \epsilon_1 \bar{E}_1) = \sigma_s$$

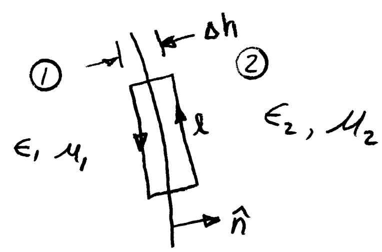
If σ_s is zero, then

$$\epsilon_2\, \hat{n} \cdot \bar{E}_2 = \epsilon_1\, n \cdot E_1$$

If the medium 1 is a conductor, then the electric flux will be zero inside the conductor and

$$\epsilon_2\, \hat{n} \cdot \bar{E}_2 = \sigma_s$$

The boundary conditions on the tangential components can be found by considering a line integral at the boundary with a leg in each medium.



The closed line integral of the electric field is equal to the negative time rate of change of the magnetic flux in the area defined by the closed line integral path. As Δ h approaches zero, the magnetic flux approaches zero. In that case

$$E_{t2}\, \ell - E_{t1}\, \ell = 0$$

where E_{t2} and E_{t1} are the tangential electric fields. Since ℓ is arbitrary

$$E_{t2} = E_{t1}$$

Vectorially this is expressed

$$\hat{n} \times (E_2 - E_1) = 0$$

The tangential component of the electric field is continuous across a boundary. If medium 1 is a conductor, then the fields must be zero inside and the external tangential electric field must be zero.

The closed line integral of the magnetic field is equal to the current enclosed plus the time rate of change of the electric flux. Again as Δh approaches zero, the electric flux approaches zero and the line integral of the magnetic field is only equal to the current enclosed. Since Δh approaches zero, the current must be a surface current. In that case

$$H_{t2}\ell - H_{t1}\ell = K_s\ell \quad \text{or} \quad H_{t2} - H_{t1} = K_s$$

Where K_s is the surface current density. Vectorially this is expressed.

$$\overline{n} \times (\overline{H}_2 - \overline{H}_1) = \overline{K}_s$$

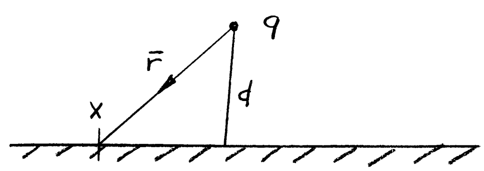
Where $\overline{K}_s$ is the vector of the current density. If medium 1 is a conductor, the magnetic field inside is zero.

$$\hat{n} \times \overline{H}_2 = \overline{K}_s$$

IMAGES

Ground planes can be handled with the method of images. The only limiting assumption is the exact solution requires an infinite ground plane. Even though we do not have an infinite ground plane, the results still give good predictions with real ground planes.

Consider a charged particle over a ground plane. From the boundary conditions

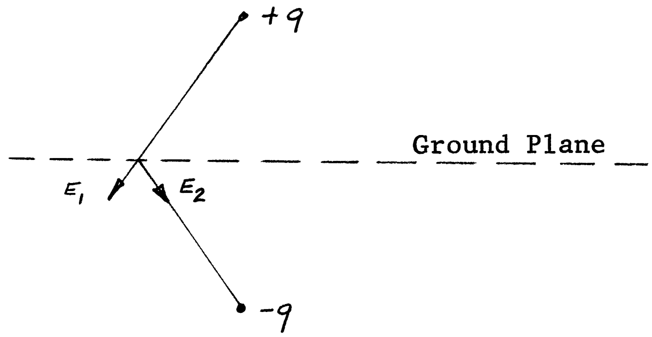


we find that the tangential electric field must be zero at the ground plane and that the electric field must be zero below the ground plane. If we consider the field at point X due to the charge, we find the field to be

$$E = \frac{q\,\hat{r}}{4\pi r^2 \epsilon}$$

where q is the charge, r is the distance, $\hat{r}$ the unit vector, and ϵ the permittivity of the medium. This field will not satisfy the boundary condition. These fields will induce a charge distribution on the ground plane which will contribute to the field and satisfy the boundary conditions. This is a nice exercise in electrostatics.

We will jump to the image solution. If we place another charged particle at the same distance, d, below the ground plane, remove the ground plane, and restrict the solution to above the ground plane, then the solution can be found.

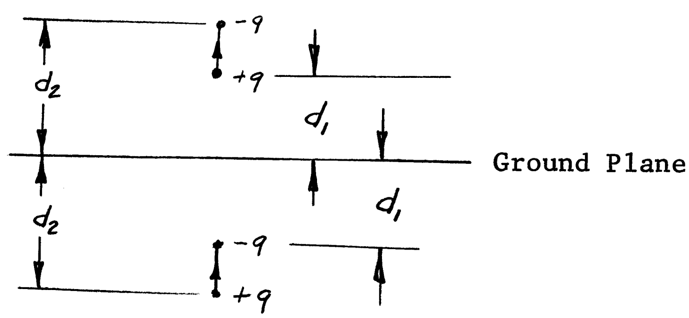


The figure above shows the solution. The fields from each charge are shown in the figure. The vector sum of these has no X or tangential component. The normal field is matched with surface currents on the ground plane.

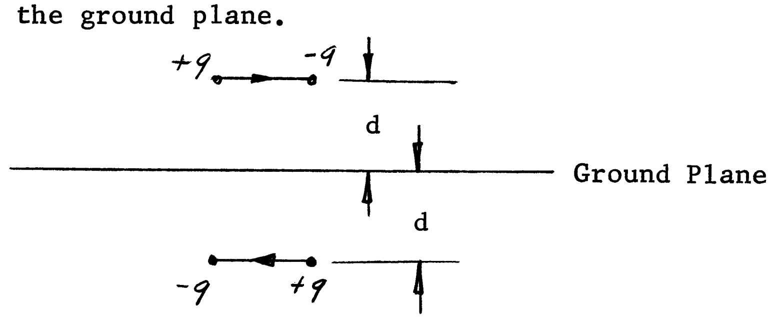
$$\rho_s = \epsilon |\bar{E}|$$

Where E is the vector sum of the two fields which is normal to the ground plane. The image charge method has solved the electrostatic problem and found the electric fields above the ground plane and found the surface currents.

If we add a distribution of charges above the ground plane, then each charge will be mirrored in the ground plane.

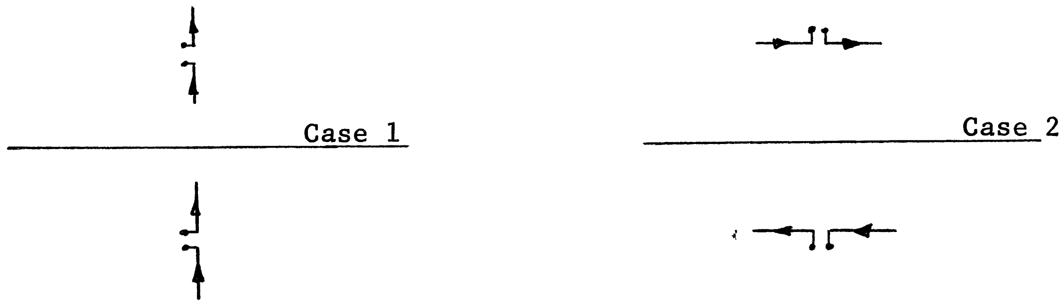


Consider two ends of a capacitor with a current between them. The above figure shows the image charges and currents in the ground plane. Vertical currents have vertical images in the ground plane. The second case is currents parallel to the ground plane.



Horizontal currents have horizontal images in the ground plane with currents in opposite directions. We can get this result by considering charges on the capacitor ends.

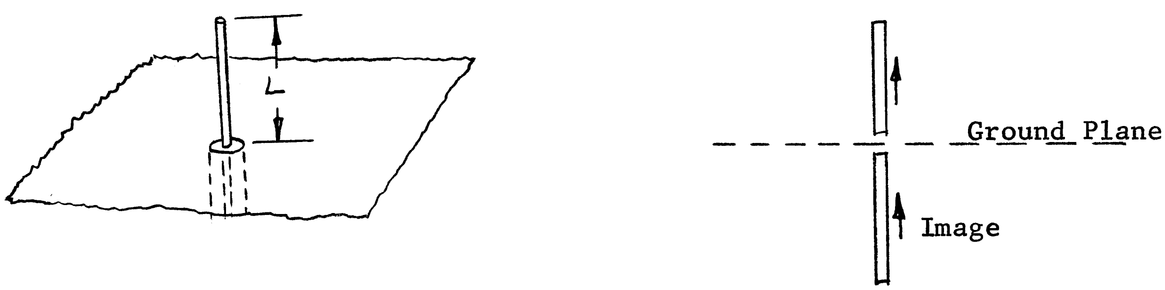
Using these results we can find the images of dipoles in a ground plane.



We can now analyze dipoles over ground planes as arrays of dipoles and ignore the fields under the ground plane.

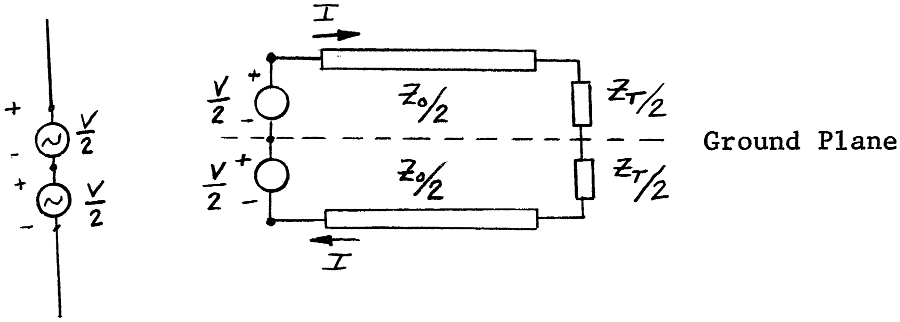
MONOPOLES

A single conductor can be fed out of a ground plane without any baluns.



The second arm of the dipole is provided by the ground plane currents. If we add the image and ignore the ground plane, then the fields can be found above the ground plane. All the results for dipoles can be carried over to monopoles except for impedance.

The impedance can be found by dividing the dipole feed into two sources. The dipole can be considered as a terminated transmission line where the



line has been shown divided into lines each with a characteristic impedance of $Z_o/2$. When combined in series these lines give the total characteristic impedance. The terminating impedance which is the equivalent radiation impedance has also been divided into two loads. The monopole antenna has a ground plane in place of the dashed line above. The impedance can be evaluated from two points of view. Ignoring the source, we can see that both the characteristic impedance and the terminating load have been halved; therefore using transmission line techniques the input impedance is one-half the value for the full dipole. In the other method we can see that the current into the monopole is the same as for the dipole but the source input voltage is one-half the value for the full dipole. Therefore the impedance which is the ratio of the input voltage to the input current is one-half the value for the full dipole.

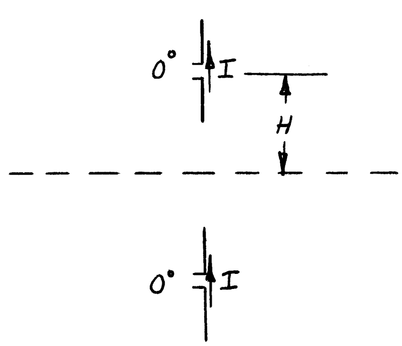
For a quarter wavelength long monopole the maximum of the pattern will be at the ground plane. This corresponds to a half-wave dipole. When the ground plane is lossy such as over the earth, the fields which propagate along the ground will be absorbed. The pattern will have a null along this direction due to the absorption. Mostly this is a low frequency effect. One solution is to bury wires or lay wires along the ground to decrease the ground impedance and give a good image.

GAIN OF MONOPOLE

The gain of a monopole is twice that for the full length dipole. We can see this in two ways. The maximum radiation intensity is the same for both the full dipole and the monopole. The input impedance of the monopole is one-half of the full dipole. Therefore the input power is only one-half that of the full dipole. Directivity (Gain) is the ratio of 4π times the maximum radiation intensity divided by the input power. Hence the gain of the monopole is twice the gain of the full dipole. The second way to look at this is to consider the integral of the radiation intensity divided into 4π times the maximum radiation intensity. The full dipole has symmetry about the angle $\theta = 90^o$ and so does the pattern. The monopole has no field beyond $\theta = 90^o$ and hence the integral of the radiation intensity is only one-half that of the full dipole. Therefore the gain is twice that of the full dipole.

DIPOLE OVER A GROUND PLANE

Vertical Dipole When a vertical dipole is over a ground plane, the antenna and its image form a two element array. The distance between the elements is 2 H and they are fed in phase. The maximum of the array (as isotropic elements) occurs at $\theta = 90^o$ which we can see by drawing a line parallel to the Z axis and considering the phase distance from each antenna. The maximum field is 2 E_o or a power radiation intensity of 4 times the power from a single radiator. To this the field of the dipole is multiplied. The maximum field of a dipole is



$$U_{MAX} = \frac{\eta |I_0|^2}{(2\pi)^2} \left(1 - \cos(\beta \frac{L}{2}) \right)^2 \qquad \theta = 90^o$$

Which is true for $L \lesssim 1.4$ from the plot on page 134. From pattern multiplication the maximum radiation intensity is 4 times the maximum radiation intensity of the dipole. The input power to the one element if found from the radiation resistance of the one element times one plus the ratio of the mutual resistance divided by the radiation resistance. From the array theory given on page 99.

$$P_{in} = R_R |I_o|^2 (1 + \frac{R_{12}}{R_R})$$

Where R_{12} is the mutual resistance between the antenna and its image. The directivity is found from the input power and the maximum radiation intensity.

$$\text{Directivity} = \frac{4\pi U_{MAX}}{R_R |I_o|^2 (1 + \frac{R_{12}}{R_R})}$$

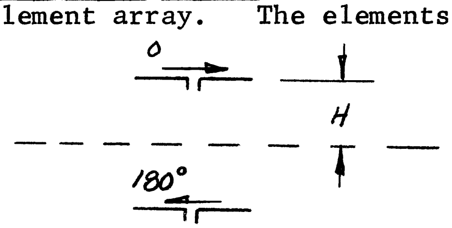
Substituting the maximum radiation intensity

$$\text{Directivity} = \frac{4\eta (1 - \text{Cos}(\beta\frac{L}{2}))^2}{(R_R + R_{12})\pi}$$

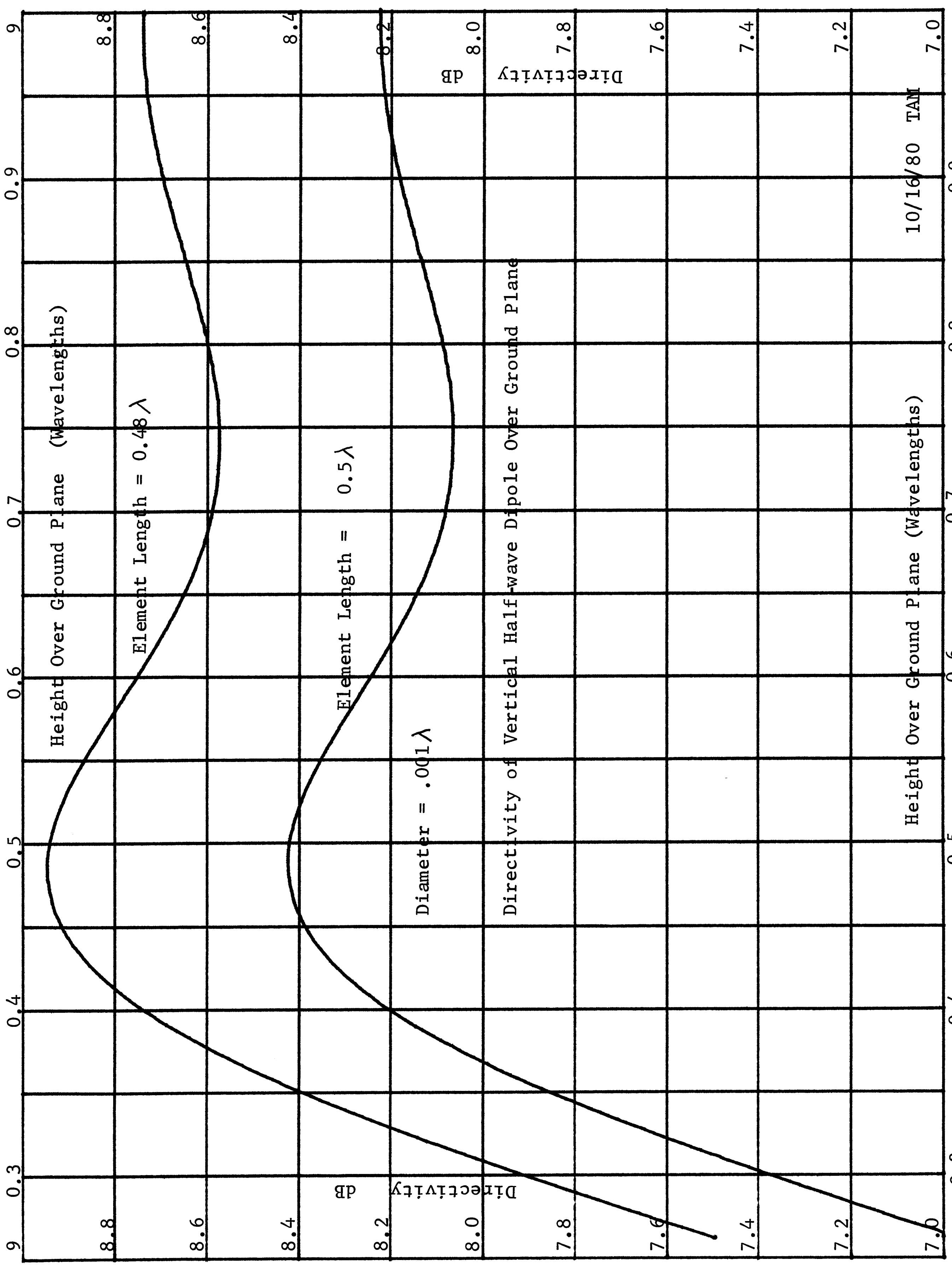
Notice that we had to only account for the power into one antenna. There is no input power into the image. On page 159 there is a plot of the gain of a "half-wave" dipole as a function of the distance over the ground plane. There are two curves plotted on the graph. A vertical dipole which has been shortened to resonate the reactance of the dipole to zero has higher gain than the full half-wave dipole. This is because the radiation resistance of the shorter dipole is less than the full dipole and the mutual resistance term changes slowly with length. In the regions where the gain peaks, the mutual resistance between the dipole and its image is negative.

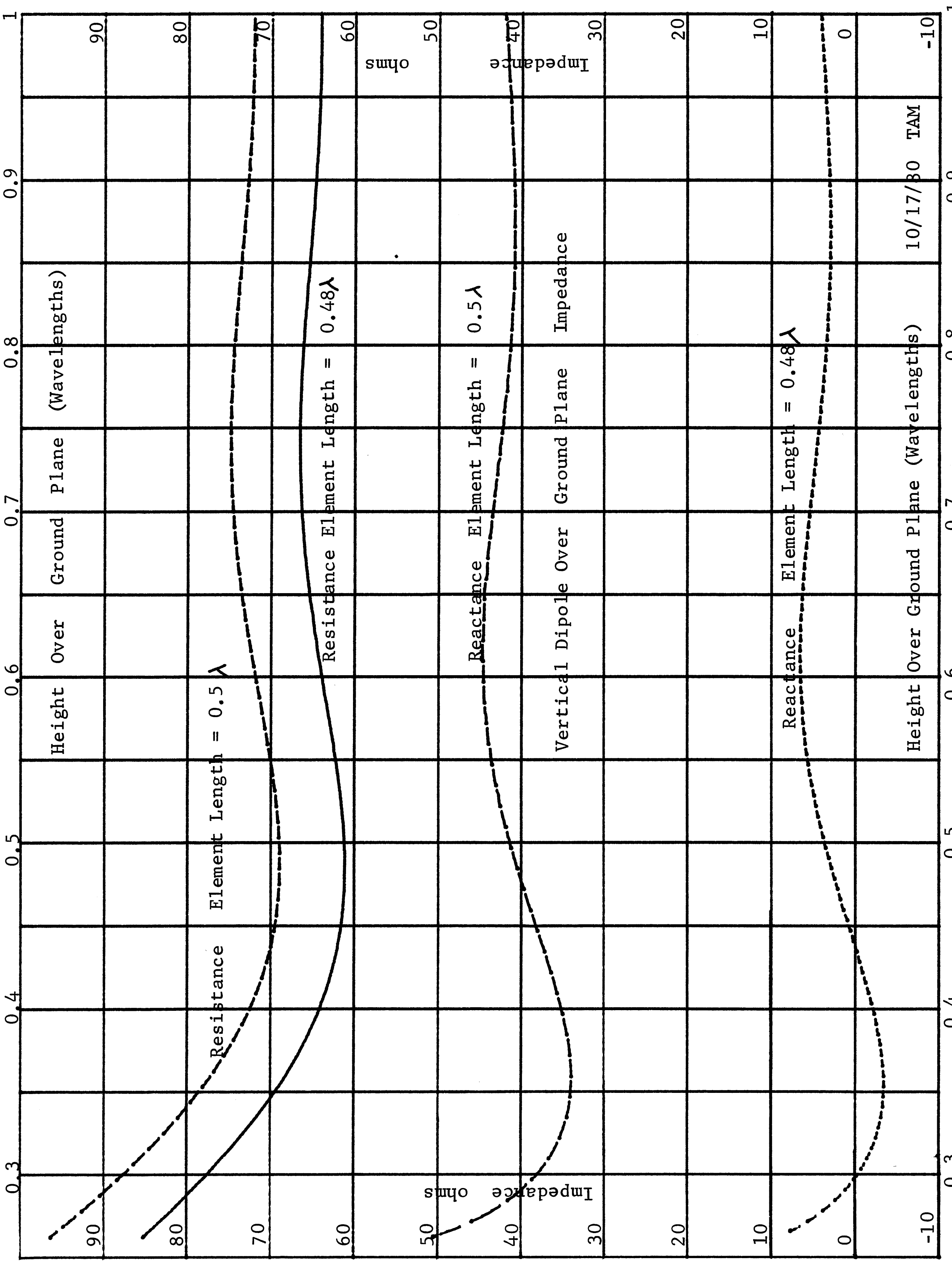
From page 98 we can see that the dipoles are being fed in the even mode. The impedance matrix gives the input impedance as $Z_{11} + Z_{12}$. On page 160 are plots of input impedance for a vertical "half-wave" dipole over a ground plane as a function of the height of the dipole over the ground plane. Both the full half-wave dipole and a resonated shortened dipole impedances are given. We can see that the input resistance of the shortened dipole is less than the full dipole which correlates with the curve on page 159 which shows higher gain for the shortened dipole. Notice how the reactance changes rapidly for changes in the length of the dipole whereas the resistance is changing slowly. In general the impedance changes slowly with changing height over the ground plane. This is because the mutual impedance term is small for dipoles spaced in this direction.

<u>Horizontal Dipole</u> - The horizontal dipole and its image also form a two element array. The elements are spaced 2 H and are fed out of phase.



This array has a null at $\theta = 90^o$. Any line drawn parallel to the Z axis to add the far field signals will see two fields 180^o out of phase or a null. The maximum of the radiation pattern occurs at $\theta = 0$ for H less than $\lambda/4$. At heights greater


Directivity of Vertical Half-wave Dipole Over Ground Plane
Element Length = 0.48 λ
Element Length = 0.5 λ
Diameter = .001 λ
Height Over Ground Plane (Wavelengths)
Directivity dB
10/16/80 TAM
0.3
0.4
0.5
0.6
0.7
0.8
0.9
7.0
7.2
7.4
7.6
7.8
8.0
8.2
8.4
8.6
8.8


Vertical Dipole Over Ground Plane Impedance
Height Over Ground Plane (Wavelengths)
Impedance ohms
Resistance Element Length = 0.5λ
Resistance Element Length = 0.48λ
Reactance Element Length = 0.5λ
Reactance Element Length = 0.48λ
10/17/80 TAM

than $\lambda/4$ the pattern splits into two or more lobes and the maximum is no longer at $\theta = 0$. One of the maximums is found from page 94.

$$\theta_{MAX} = \cos^{-1}\left(\frac{\lambda}{4H}\right) \qquad H \geq \lambda/4$$

The normalized maximum radiation intensity of a two element isotropic array with phases 0 and 180° is given on page 93.

$$U_{A\ MAX} = 4 \sin^2\left(\frac{2\pi H}{\lambda}\right) \qquad H \leq \lambda/4$$

$$= 4 \qquad H \geq \lambda/4$$

To this we multiply the maximum radiation intensity of the dipole given on page 132.

$$U_{d\ MAX} = \frac{\eta |I_o|^2}{(2\pi)^2}\left(1 - \cos\left(\beta\frac{L}{2}\right)\right)^2 \qquad \theta = 0 \text{ for } L \leq 1.4\lambda$$

Where L is the dipole length and η impedance of free space. Like the vertical dipole the input power is found from the radiation resistance and the mutual resistance.

$$P_{in} = R_R |I_o|^2 \left(1 - \frac{R_{12}}{R_R}\right)$$

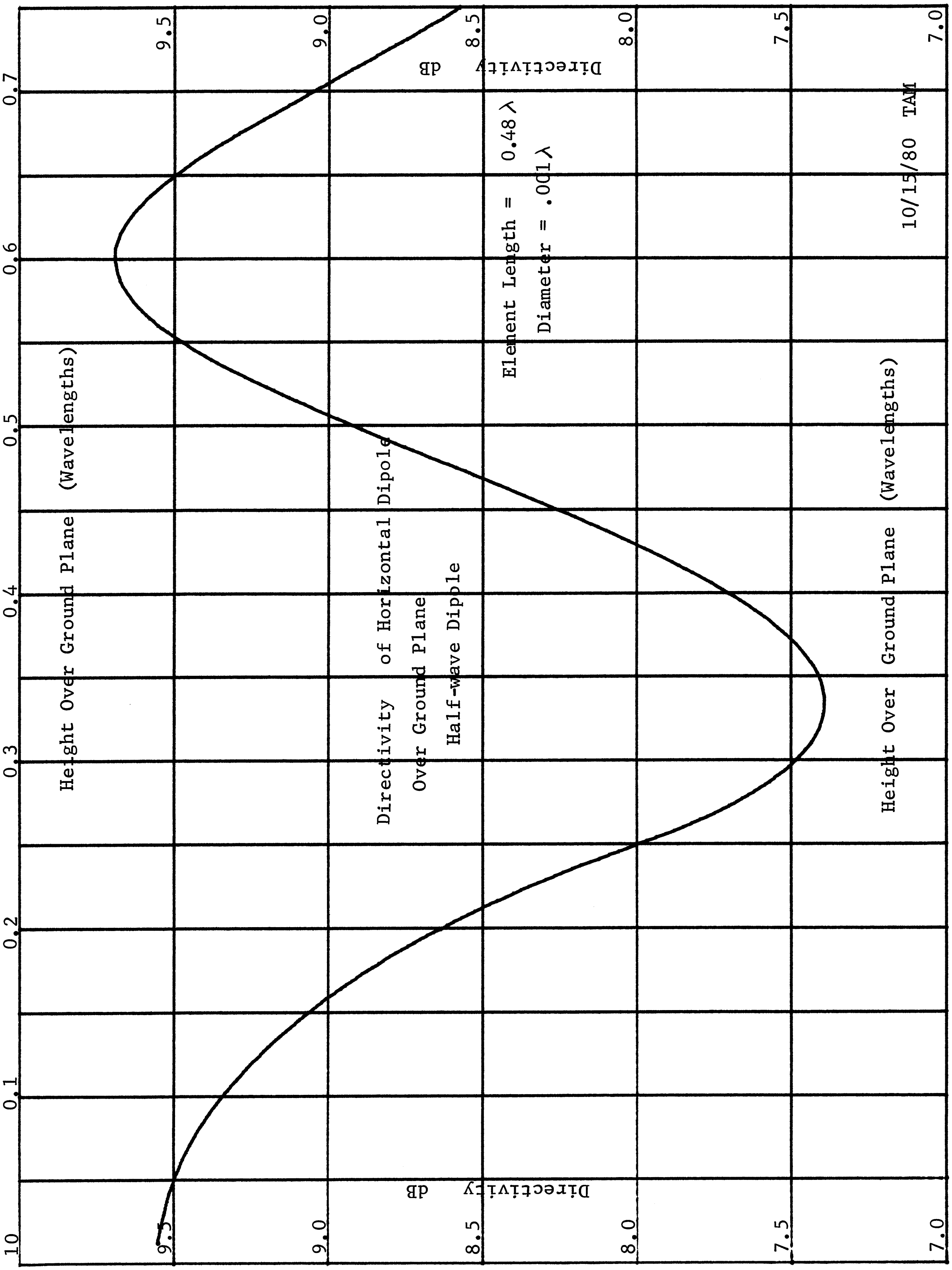
Where R_{12} is the mutual resistance between the antenna and its image. In this case the antennas are fed in the odd mode. Using this input power to the single antenna element, and the product of the normalized radiation intensity of the array and the dipole pattern, the directivity can be found.

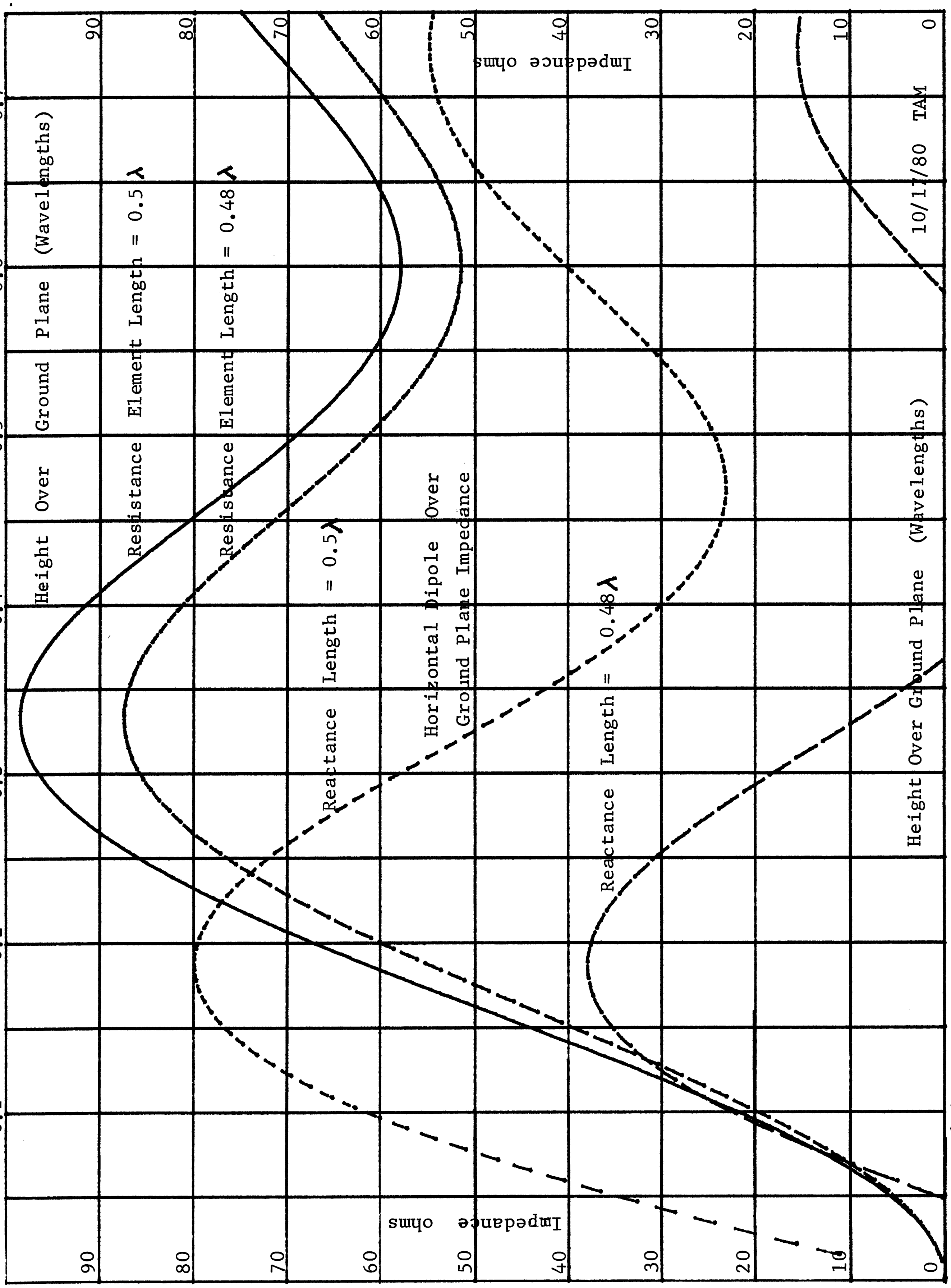
$$\text{Directivity} = \frac{4 \sin^2\left(\frac{2\pi H}{\lambda}\right) \eta \left(1 - \cos\left(\beta\frac{L}{2}\right)\right)^2}{(R_R - R_{12})} \qquad H \leq \lambda/4$$

$$= \frac{4\eta\left(1 - \cos\left(\beta\frac{L}{2}\right)\right)^2}{\pi(R_R - R_{12})} \qquad H \geq \lambda/4$$

This function is plotted on page 162 for a resonated "half-wave" dipole versus the ground plane spacing. This plot is similiar to the curve of directivity for the odd mode two element isotropic array given on page 95. The gain peaks up for close ground plane spacings. The actual directivity is dependent on the length of the dipole like the vertical dipole over a ground plane.

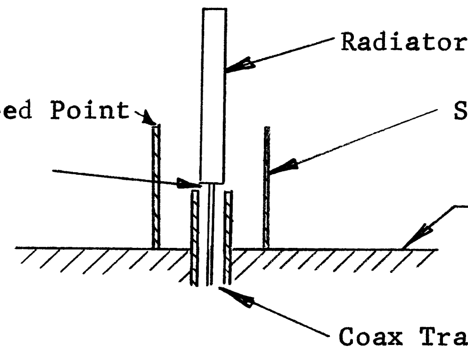
The dipole and its image are being fed in the odd mode. The impedance matrix gives the input impedance as $Z_{11} - Z_{12}$. On page 163 is a plot of the horizontal resonated half-wave dipole antenna input impedance as a function of the height of the dipole over ground. Both the full half-wave dipole and the shortened dipole impedances are given. The resistive portion of the impedance changes slowly but the reactance rapidly for changes in length. Notice that the impedance decreases rapidly for close ground spacings which will give a poor efficiency antenna.


Directivity of Horizontal Dipole
Over Ground Plane
Half-wave Dipole
Height Over Ground Plane (Wavelengths)
Directivity dB
Element Length = 0.48 λ
Diameter = .001 λ
10/15/80 TAM
0.1
0.2
0.3
0.4
0.5
0.6
0.7
7.0
7.5
8.0
8.5
9.0
9.5

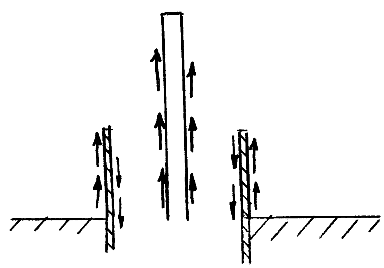

Height Over Ground Plane (Wavelengths)
Resistance Element Length = 0.5 λ
Resistance Element Length = 0.48 λ
Reactance Length = 0.5 λ
Horizontal Dipole Over
Ground Plane Impedance
Reactance Length = 0.48 λ
Height Over Ground Plane (Wavelengths)
10/17/80 TAM
Impedance ohms
Impedance ohms
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0
10
20
30
40
50
60
70
80
90

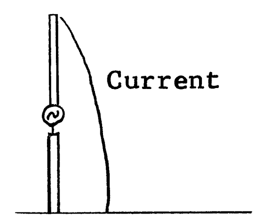
SLEEVE ANTENNA

The bandwidth of a dipole antenna or a monopole antenna may be increased by using thicker elements. The curves on page 133 of the symmetrical dipole show that the reactance slope at resonance decreases as the diameter of the elements increases. But the resistive part of the impedance does not change much with an increase in diameter. The sleeve antenna solves some of this problem by moving the effective feed point of the antenna. Below is a diagram of a general sleeve monopole antenna.



The sleeve around the antenna shields the currents in the lower portion of the monopole from radiating and moves the virtual feed point of the antenna to the top of the sleeve. The current on the radiator and the sleeve are given in the diagram below.

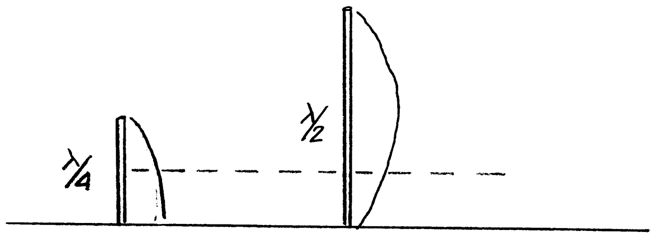




The current flows up the center conductor of the coax structure in the sleeve region and continues up the radiator. The current on the sleeve is flowing down the inner surface of the sleeve to match the coaxial currents on the center radiator and it is flowing up the outer surface of the sleeve to match the current flowing down the inner surface. Since the currents inside the sleeve are shielded, only the current on the radiator and the outer surface of the sleeve radiate. The effective radiation current of the antenna is shown in the diagram above to the right and is the usual approximately sinusoidal standing wave current of the monopole. The far field pattern of the monopole is only slightly effected by the sleeve.

The sleeve has the effect of moving the feed point to the end of the sleeve. We can use this to reduce the variation of the standing wave current at the feed point. Consider a quarter wavelength and a half wavelength monopoles. The current patterns are given in the diagram on page 163b. The quarterwave monopole will have a low impedance feed point at the base where the current

163a

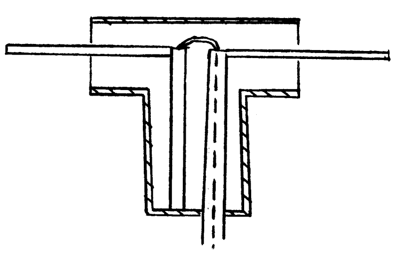


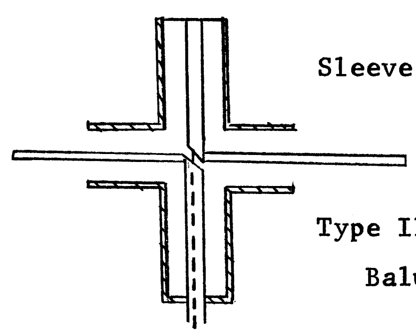
is high. The half-wave monopole has a high impedance feed point at the base where the standing wave current is low. If we would feed the two antennas at the location shown by the dashed line in the diagram above, then the standing wave current would be the same in both antennas at the feed point and the input resistance of the antennas would be approximately the same. This effect will broadband the antenna feed. The diagram shows two different antennas with the feed point located at a different point when we consider the ratio of the feed point distance from the ground plane to the total length of the antenna. When designing a single antenna to cover both lengths at different frequencies, a compromise must be made and there will be a variation in the feed point current. It is possible to achieve at least an octave bandwidth from the sleeve antenna.

In the first diagram on page 163a a more general feed network is shown for the sleeve monopole. The portion of the antenna inside the sleeve is shielded from radiating and can be used to match the antenna. The impedance of the coaxial line between the sleeve and the radiator can be varied to transform the impedance at the virtual antenna feed point at the end of the sleeve to another impedance at the input to the coax feed point. In this diagram the coax feed line has been extended beyond the ground plane. Between the outer shield of the coax feed line and the sleeve is formed another coaxial line. This coax line is shorted by the ground plane. At the feed point of the coax this line is a series shorted stub which may be used to resonate out reactance transformed from the antenna feed point down the sleeve and radiator transmission line. Similiar to the shunt fed antenna, this antenna is usually designed empirically.

The dipole can also be designed with a sleeve. We analyze the monopole antenna by the method of images and we can just image the sleeve as well into a dipole.

Sleeve Dipole with Folded Balun



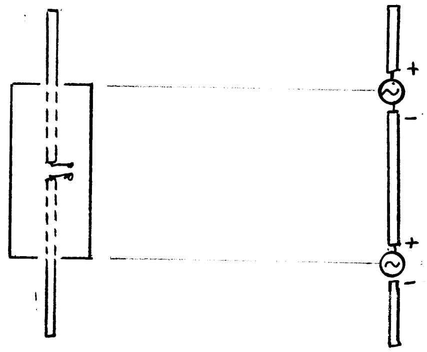


Sleeve Dipole

Type II Sleeve Balun

Above are two diagrams of the sleeve dipole with different baluns. Like the monopole, the antenna is designed empirically to achieve a broadband impedance match. The pattern of the antenna to a first order approximation is the same as the center fed antenna.

The sleeve on the dipole is equivalent to feeding the antenna at two points.



Because the antenna is fed symmetrically, there will not be the problems of the asymmetrically fed antenna as discussed on page 136 with a full wavelength dipole.

The sleeve on the antenna does not have to fully cover the center radiator. The dipole can be made into a sleeve antenna by adding strips or rods on both sides of the center dipole as shown below.

The antenna above is fed with a folded balun. The center radiator forms a three wire transmission line with the two side rods with the two outer ones at the same potential. In this case it is necessary that the two lines forming the sleeve be close to the center conductor so that the currents from the transmission line section do not radiate significantly. This technique is handy at low frequencies where the dipole is long and a sleeve would greatly increase the weight and wind loading of the antenna.

1. Bock, E. L., J. A. Nelson, and A. Dorne, Very High Frequency Techniques, Chapter 5, Radio Research Labortary Staff, McGraw Hill, New York, 1947.

2. Kraus, J. D., Antennas, pp. 422, McGraw Hill, New York, 1950.

3. Poggio, A. J. and P. E. Mayes, "Pattern bandwidth optimization of the sleeve monopole antenna", IEEE Trans. on Antennas and Propagation, Vol. AP-14, pp. 643-645, Sept. 1966

4. Stutzman, W. L. and G. A. Thiele, Antenna Theory and Design, pp.278, Wiley, New York, 1981.

TRAVELING WAVE CURRENTS

We have considered wire antennas with standing waves on the conductors which arise from the termination of the antenna as a transmission line. If the end of the antenna is not an open circuit or a short circuit but a load, then there are not two traveling waves going in opposite directions but in one direction only. For a traveling wave current along the Z axis, the current is

$$I = I_0 e^{-j\beta p z} \qquad \beta = \frac{2\pi}{\lambda}$$

where p is a constant which is the relative propagation of the current on the wire ($p \gtrsim 1$). In most cases p is one or the waves travel at the speed of light in free space. To find the radiated fields we analyze this as a linear continuous array. The magnetic vector potential in the far field approximation is

$$A_z = \frac{I_0 e^{-j\beta r}}{4\pi} \int_{-L/2}^{L/2} e^{-j\beta z(p-\cos\theta)} dz$$

The result of this integral is

$$A_z = \frac{I_0 e^{-j\beta r} \sin\left(\beta \frac{L}{2}(p-\cos\theta)\right)}{2\pi r \beta (p-\cos\theta)}$$

From this the electric field is found $\bar{E} = -j\omega\mu\bar{A}$. Note that we will want the θ component: $E_\theta = \bar{a}_\theta \cdot \bar{a}_z E_z$ and that $\bar{a}_\theta \cdot \bar{a}_z = -\sin\theta$. Therefore $E_\theta = j\omega\mu \sin\theta A_z$. The electric field is given by

$$E_\theta = \frac{j\omega\mu I_0 e^{-j\beta r}}{2\pi r \beta (p-\cos\theta)} \sin\left(\beta \frac{L}{2}(p-\cos\theta)\right) \sin\theta$$

The electric field expression can be reduced by noting: $\beta = \omega\sqrt{\mu\epsilon}$ $\quad \eta = \sqrt{\frac{\mu}{\epsilon}}$

$$E_\theta = \frac{j\eta I_0}{2\pi r} e^{-j\beta r} \sin\theta \left[\frac{\sin\left(\beta \frac{L}{2}(p-\cos\theta)\right)}{(p-\cos\theta)}\right]$$

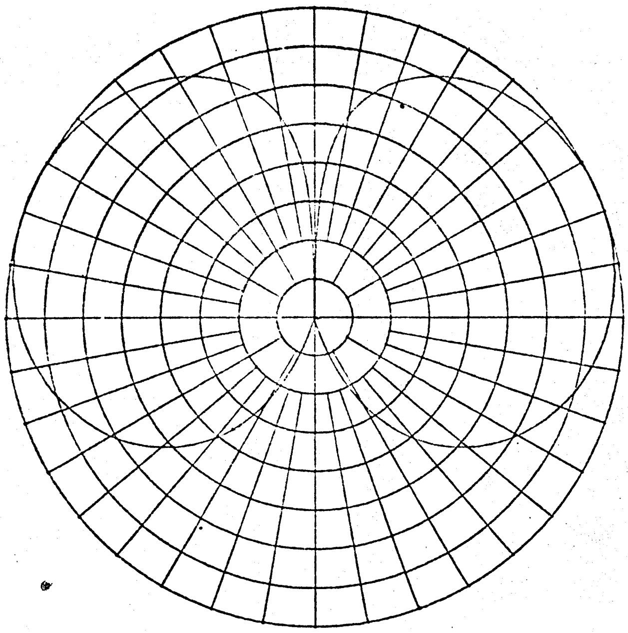
The magnetic field of the spherical wave is found from $H_\phi = E_\theta/\eta$

The field has a null on the Z axis like the dipole due to the Sin term. The Sin X/ X term has a maximum at $\theta = 0$ for p = 1 but the pattern has a null at that point. On pages 165 and 166 are patterns of a traveling current wave on the Z axis. The Z axis corresponds to $\theta = 0$ with the radiation having symmetry about the Z axis which can be seen from rotating the antenna about the Z axis. As the length of the antenna is increased the pattern becomes more directive and approaches the positive Z axis. The current is traveling from the negative Z axis towards the positive Z axis.

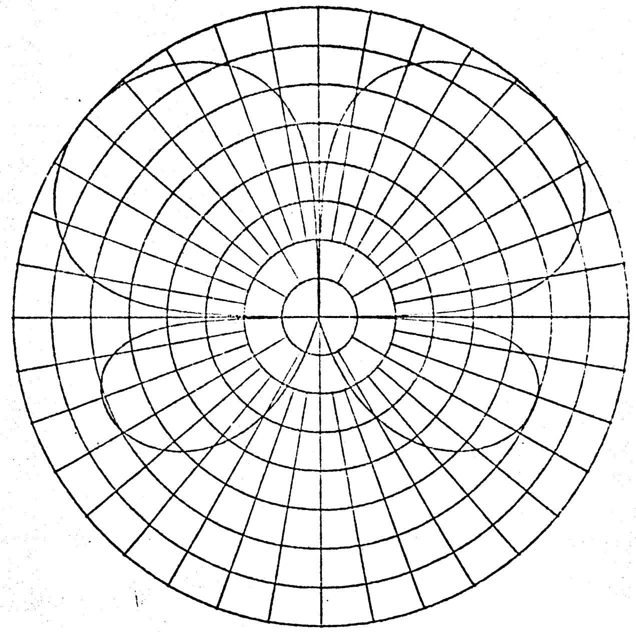
The radiation intensity is found from $E_\theta H_\phi^* r^2$

$$U = \frac{\eta |I_0|^2}{(2\pi)^2} \sin^2\theta \left[\frac{\sin\left(\beta \frac{L}{2}(p-\cos\theta)\right)}{p-\cos\theta}\right]^2$$

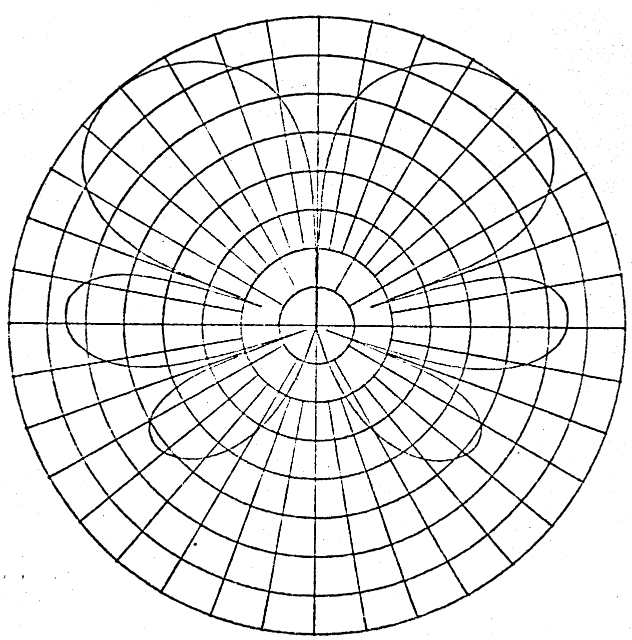
TRAVELING CURRENT WAVE HALF-WAVELENGTH LONG



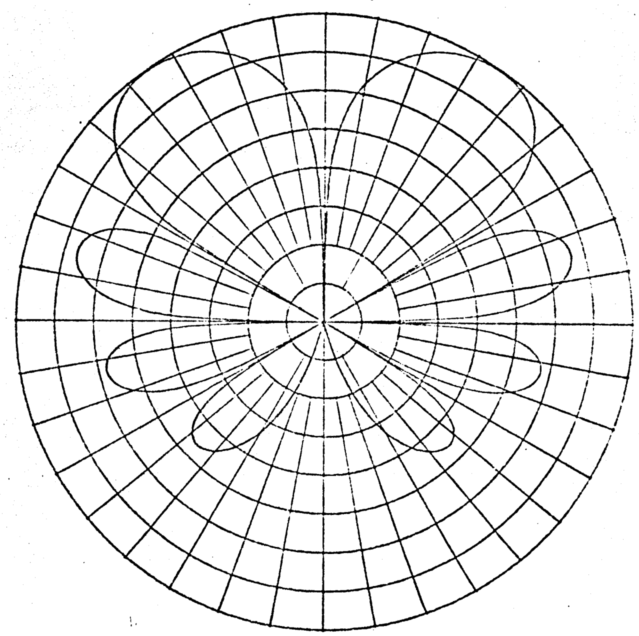
TRAVELING CURRENT WAVE ONE WAVELENGTH LONG



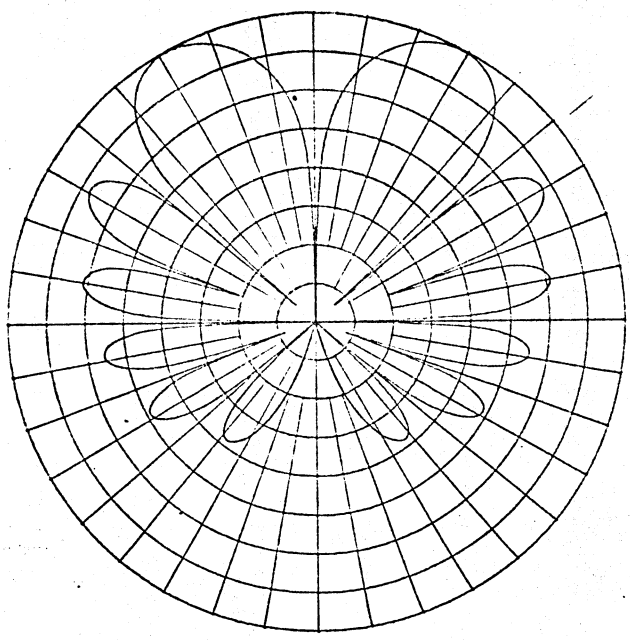
TRAVELING CURRENT WAVE ONE AND HALF WAVELENGTHS LONG



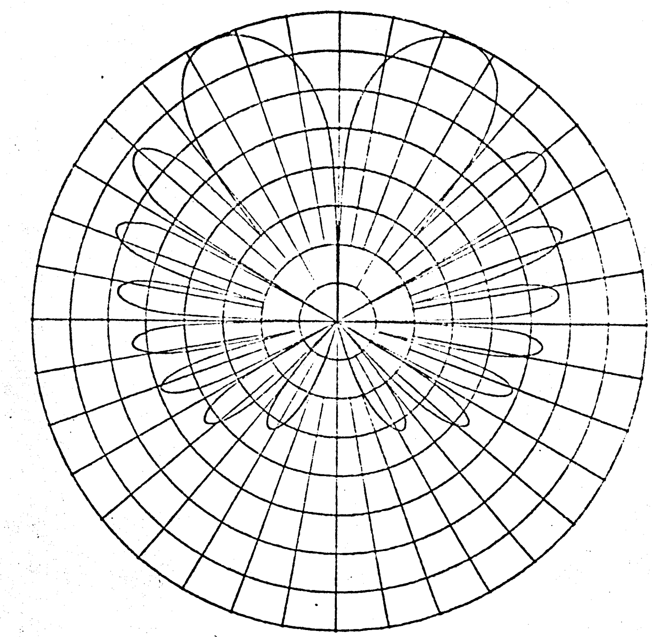
TRAVELING CURRENT WAVE TWO WAVELENGTHS LONG



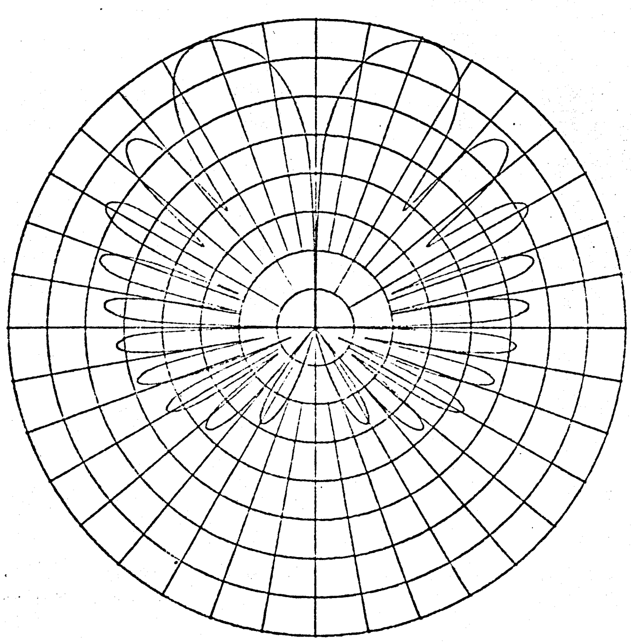
TRAVELING CURRENT WAVE THREE WAVELENGTHS LONG



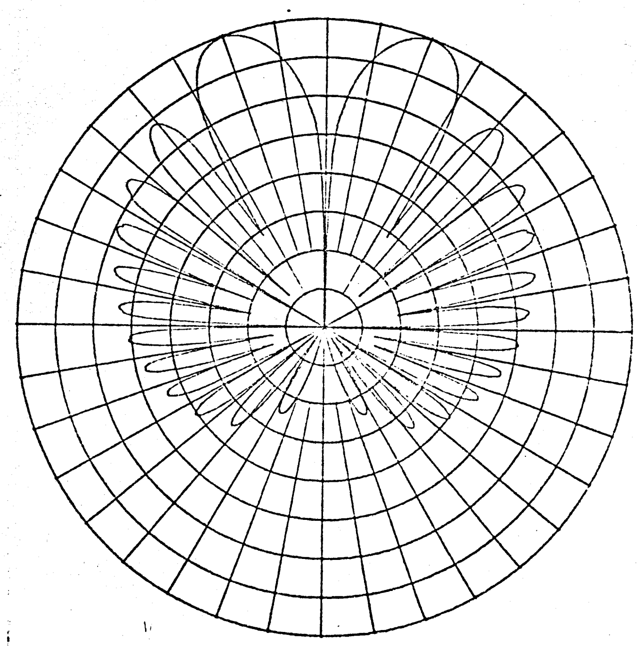
TRAVELING CURRENT WAVE FOUR WAVELENGTHS LONG



TRAVELING CURRENT WAVE FIVE WAVELENGTHS LONG



TRAVELING CURRENT WAVE SIX WAVELENGTHS LONG



The total radiated power is the surface integral of the radiation intensity.

$$P_r = \frac{\eta |I_0|^2}{2\pi} \int_0^{\pi} \sin^3\theta \left[\frac{\sin\left(\beta \frac{L}{2}(p - \cos\theta)\right)}{p - \cos\theta} \right]^2 d\theta$$

The radiation resistance is found from the total radiated power.

$$R_r = \frac{P_r}{|I_0|^2}$$

$$R_r = \frac{\eta}{2\pi} \int_0^{\pi} \sin^3\theta \left[\frac{\sin\left(\beta \frac{L}{2}(p - \cos\theta)\right)}{p - \cos\theta} \right]^2 d\theta$$

Like the dipole, the radiation resistance is given as an integral. A curve of the radiation resistance versus length is given on page 168. There are no resonances in the radiation resistance response and it increases with length as would be expected.

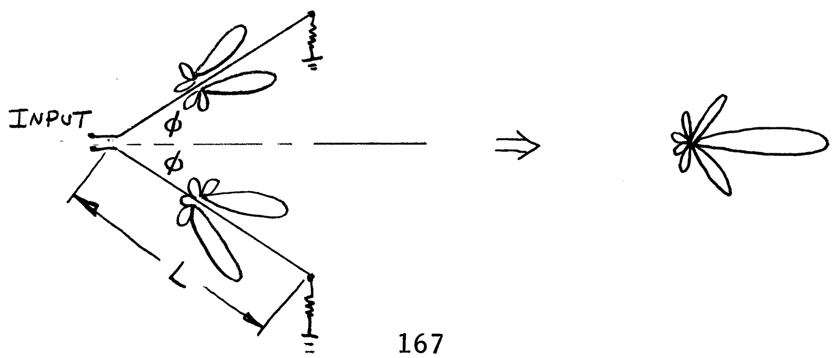
As with the dipole, the radiation resistance is referenced to the maximum current. The current magnitude is the same everywhere on the line so the radiation resistance is also the resistance at the input. All this is complicated by the load on the end of the antenna. In general the radiation resistance will not give the input resistance. Any antenna of this type will have poor efficiency because power is dissipated in the termination.

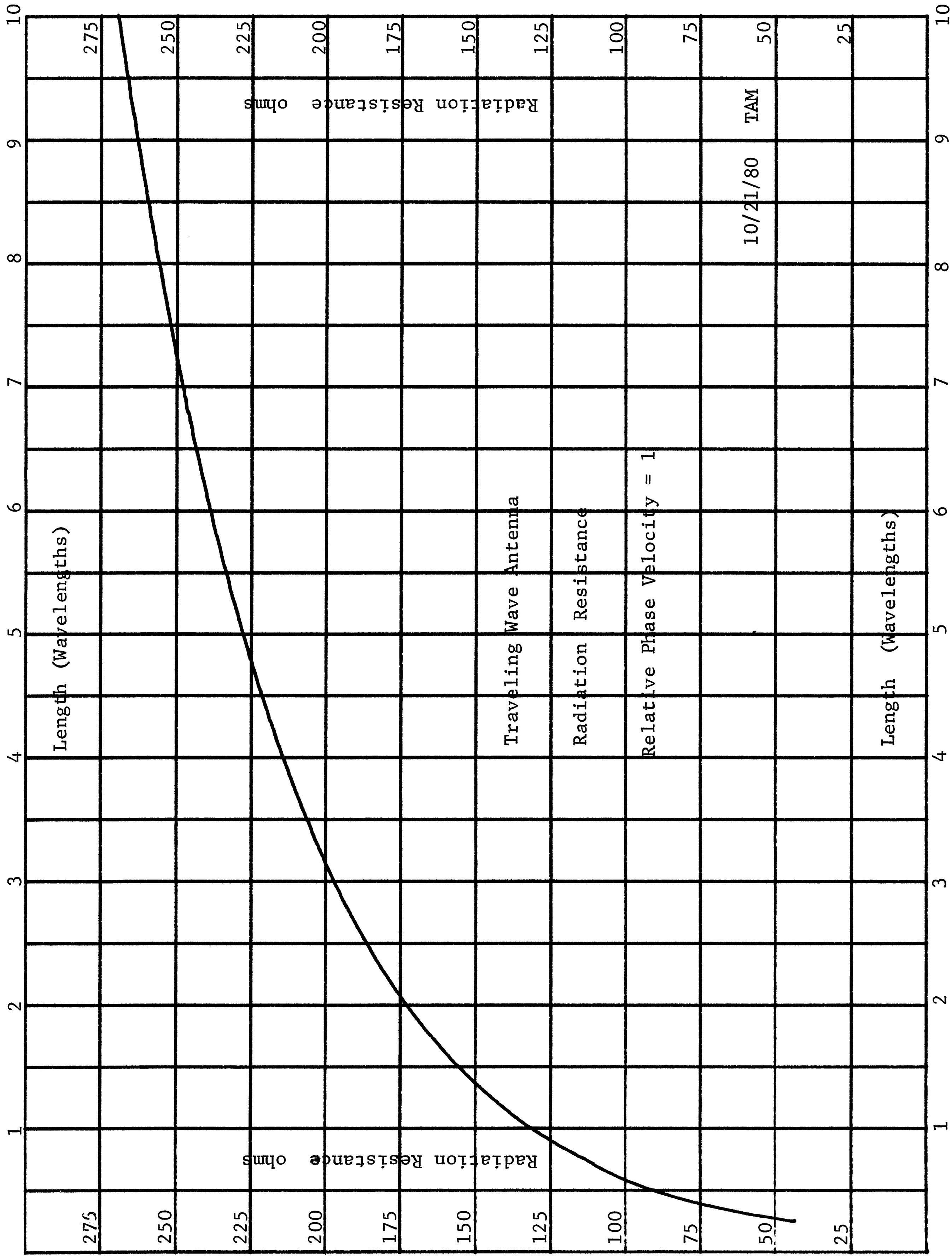
A plot of the maximum of the beam is given on page 169. The beam gets closer to the axis of the wire as the length is increased. After the length gets to eight or nine wavelengths, the beam direction changes slowly.

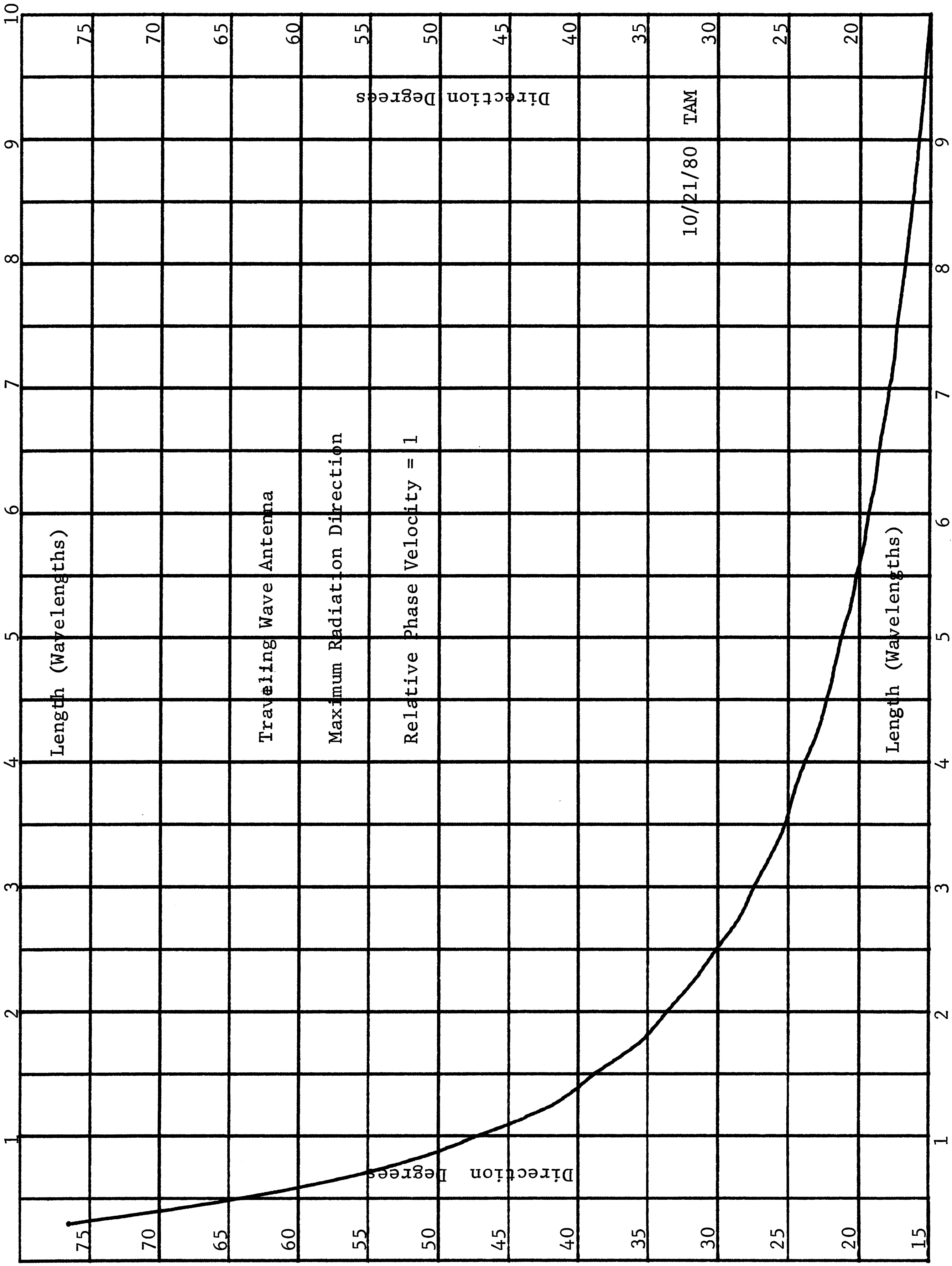
A plot of the directivity of the traveling current wave antenna is given on page 170. As can be seen from the plots on page 165 and 166, the directivity increases with increasing length of the antenna. These antennas are used when it is important to have good directivity but not necessarily good gain.

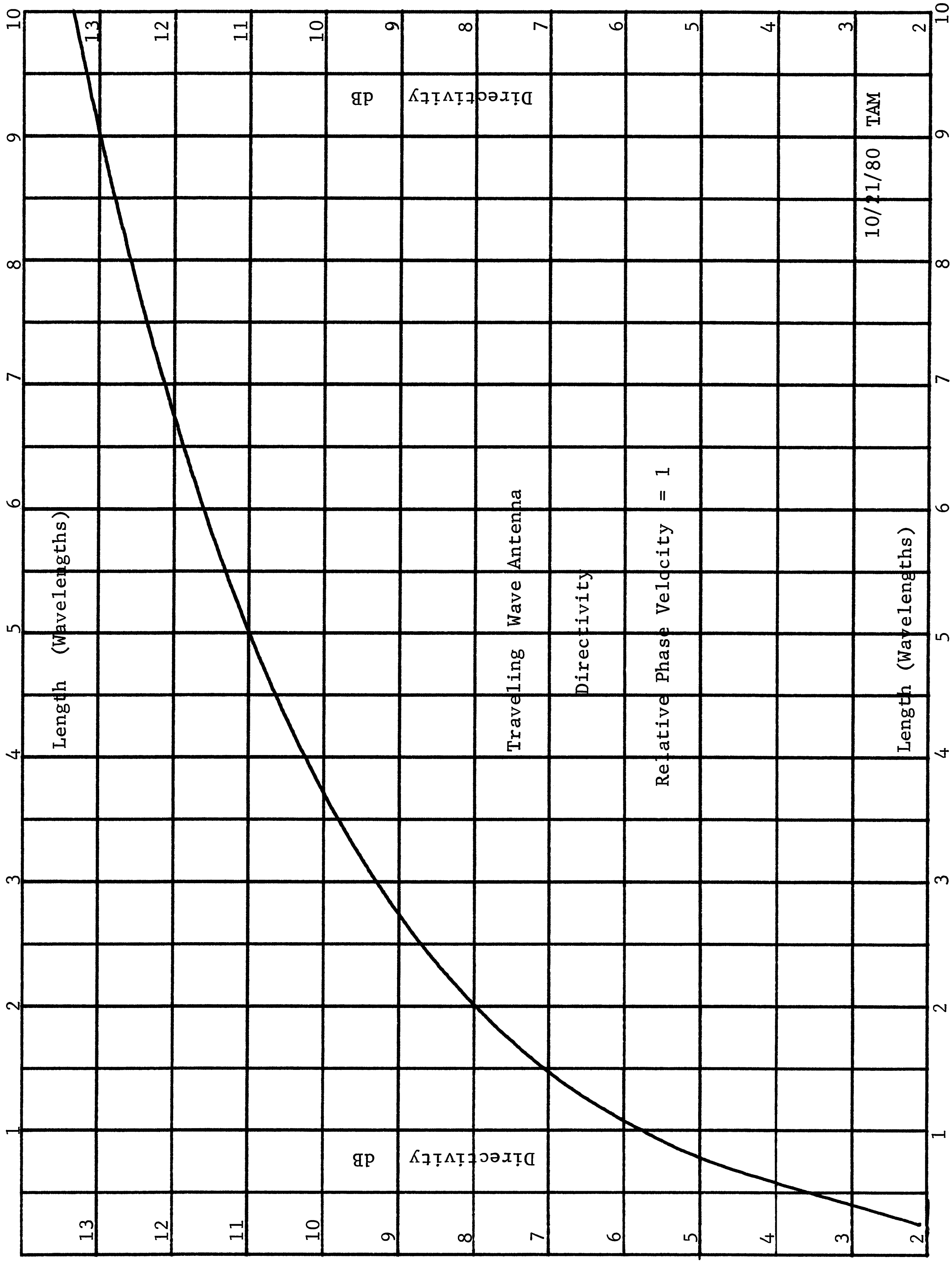
VEE ANTENNA

If we combine two traveling wave antennas into a vee structure, we can combine the two offset beams into a single beam and get some degree of cancellation of the other two beams.




Traveling Wave Antenna
Radiation Resistance
Relative Phase Velocity = 1
Length (Wavelengths)
Radiation Resistance ohms
10/21/80 TAM
1
2
3
4
5
6
7
8
9
10
25
50
75
100
125
150
175
200
225
250
275


Travelling Wave Antenna
Maximum Radiation Direction
Relative Phase Velocity = 1
Length (Wavelengths)
Direction Degrees
10/21/80 TAM

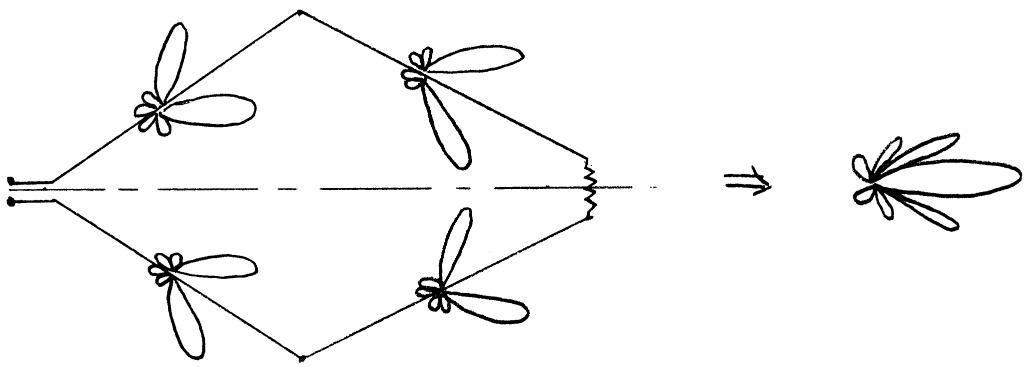


Traveling Wave Antenna
Directivity
Relative Phase Velocity = 1
Length (Wavelengths)
Length (Wavelengths)
Directivity dB
Directivity dB
10/21/80 TAM

The angle between the two arms is picked so that the beams line up. This angle can be found from the graph on page 169 which is the angle off the wire axis of the beam. This antenna can be used over a band of frequencies but the two beams will not line up except at the center frequency.

RHOMBIC ANTENNA

Two Vee antennas can be combined into a rhombic structure and a single termination load used between the arms. This eliminates the need to have a load



connected to ground. When the antenna is fed from balanced line, there is a virtual short half way through the termination load. Like the Vee antenna the angle between the arms is picked to maximize the radiation on the axis of the antenna. These antenna are usually supported on towers at the corners with a system of ground radials to reduce the impedance of the ground under the antenna. In this configuration the pattern will resemble the horizontal dipole over a ground plane. There is a null in the pattern at boresight and the beam points up into the air and generates sky waves.

The antenna beam will be at an angle of α with respect to horizontal when the antenna is a distance H above the ground. This height is given by:

$$H = \frac{\lambda n}{4 \sin \alpha} \qquad n = 1, 3, 5, \ldots$$

The antenna has good directivity but poor efficiency. This antenna is used in the 1 to 50 MHz band mostly.