MUTUAL IMPEDANCE

The mutual impedance matrix was introduced on page 52 when we discussed reciprocity which stated that the transmission between any two antennas made of isotropic materials was the same regardless of which antenna is transmitting. This was proved using the network result that the mutual impedance matrix is symmetrical. A general impedance matrix for N inputs has the result:

$$Z_{ij} = Z_{ji}$$

For a general set of antennas we get a set of linear equations.

$$\begin{aligned}
V_1 &= Z_{11}I_1 + Z_{12}I_2 + \cdots + Z_{1N}I_N \\
V_2 &= Z_{21}I_1 + Z_{22}I_2 + \cdots + Z_{2N}I_N \\
&\vdots \\
V_N &= Z_{N1}I_1 + Z_{N2}I_2 + \cdots + Z_{NN}I_N
\end{aligned}$$

Z_{ii} is the self impedance of the antenna with no other antennas present and Z_{ij} is the mutual impedance between the antennas. When we apply the reciprocity theorem, the matrix becomes symmetric

$$\begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ \vdots \\ V_N \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} & \cdots & Z_{1N} \\ Z_{12} & Z_{22} & \cdots & Z_{2N} \\ Z_{13} & Z_{23} & \cdots & Z_{3N} \\ \vdots & \vdots & & \vdots \\ Z_{1N} & Z_{2N} & \cdots & Z_{NN} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ \vdots \\ I_N \end{bmatrix}$$

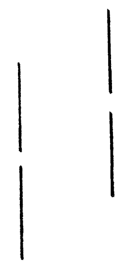
We can state some general rules about mutual impedances. The mutual impedance is a measure of the coupling between two antennas. The transmission between the input of one antenna and the output of the other is proportional to $|Z_{12}|^2$. As the distance between two antennas increases the transmission decreases and the mutual impedance decreases. The pattern response also effects the coupling and mutual impedance. Broadside coupled dipole elements have larger mutual impedances than colinear since in the far field the colinear elements do not couple. The mutual impedance terms do not fall off uniformly but cycle.

Dipole Mutual Impedance

The most used mutual impedance is that between wire elements. The mutual impedance can be derived from a method of P. S. Carter[1] called the induced EMF method. This method assumes a sinusoidal current on each dipole and

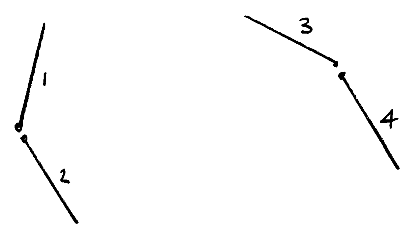
1 Carter, P.S. (1932) Circuit Relations in Radiating Systems and Applications to Antenna Problems, Proc. IRE 20, 1004

calculates the induced voltage on one antenna when the other is fed. Kraus[2] presents an extensive discussion of the method. The equations for equal length dipoles are given in Kraus[2] and Hansen[3]. Curves for the broadside and colinear coupled dipoles each a half-wave long are given on pages 309 and 310. We can see from the curves that the broadside coupled dipoles have larger mutual impedances than the colinear case. There are also published equations for dipoles of unequal lengths in echelon[4].



Echelon Dipoles

Richmond[5] has published general equations for monopoles with skewed orientation with sinusoidal current distributions. We can find the mutual impedance between Vee dipoles by taking the sum of four monopole to monopole mutual impedance terms.



The mutual impedance between the two Vee dipoles above is given by

$$\text{Mutual Impedance} = Z_{13} + Z_{14} + Z_{23} + Z_{24}$$

where Z_{ij} are monopole mutual impedances.

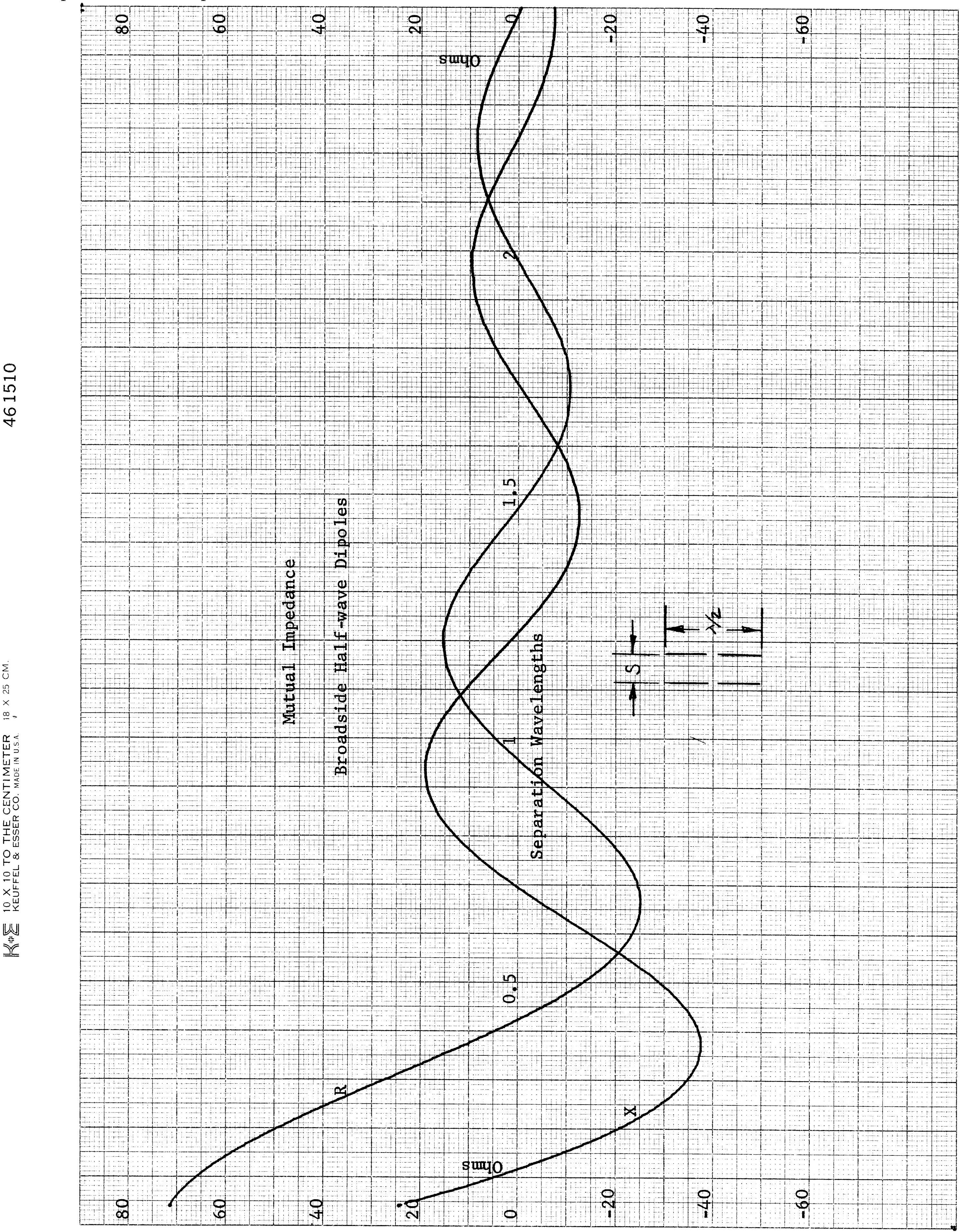
On pages 311 and 312 are results for dipoles of unequal length with one dipole a half-wavelength long. Notice on page 311 that the mutual impedance between two antennas approaches that of a single dipole as the spacing approaches zero. This result is used to find the approximate input impedance of a thick dipole. The impedance is equal to the mutual impedance of two dipoles separated by the radius.

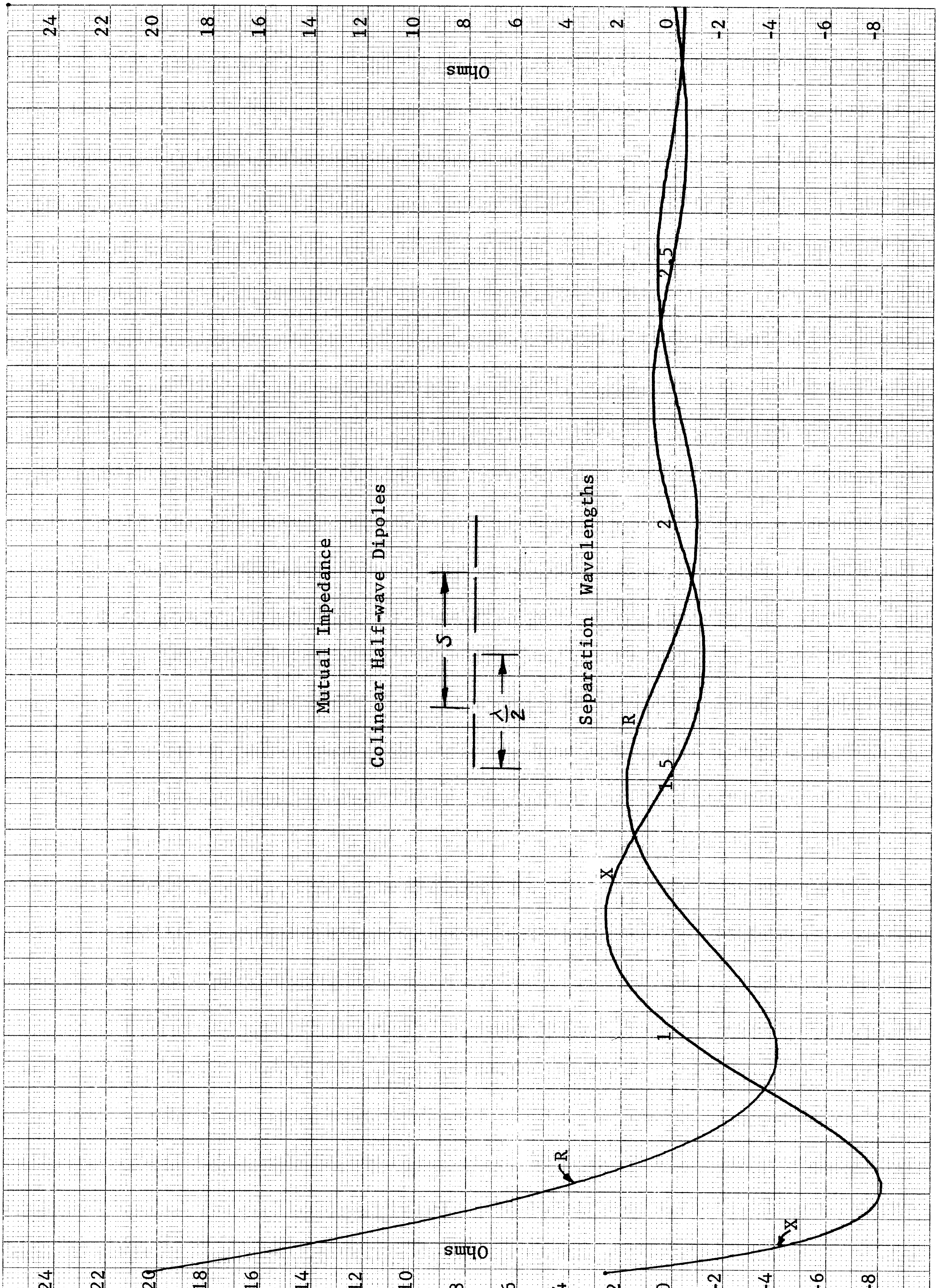
2. Kraus, J. D., (1950) Antennas , McGraw Hill, New York, pp. 254.

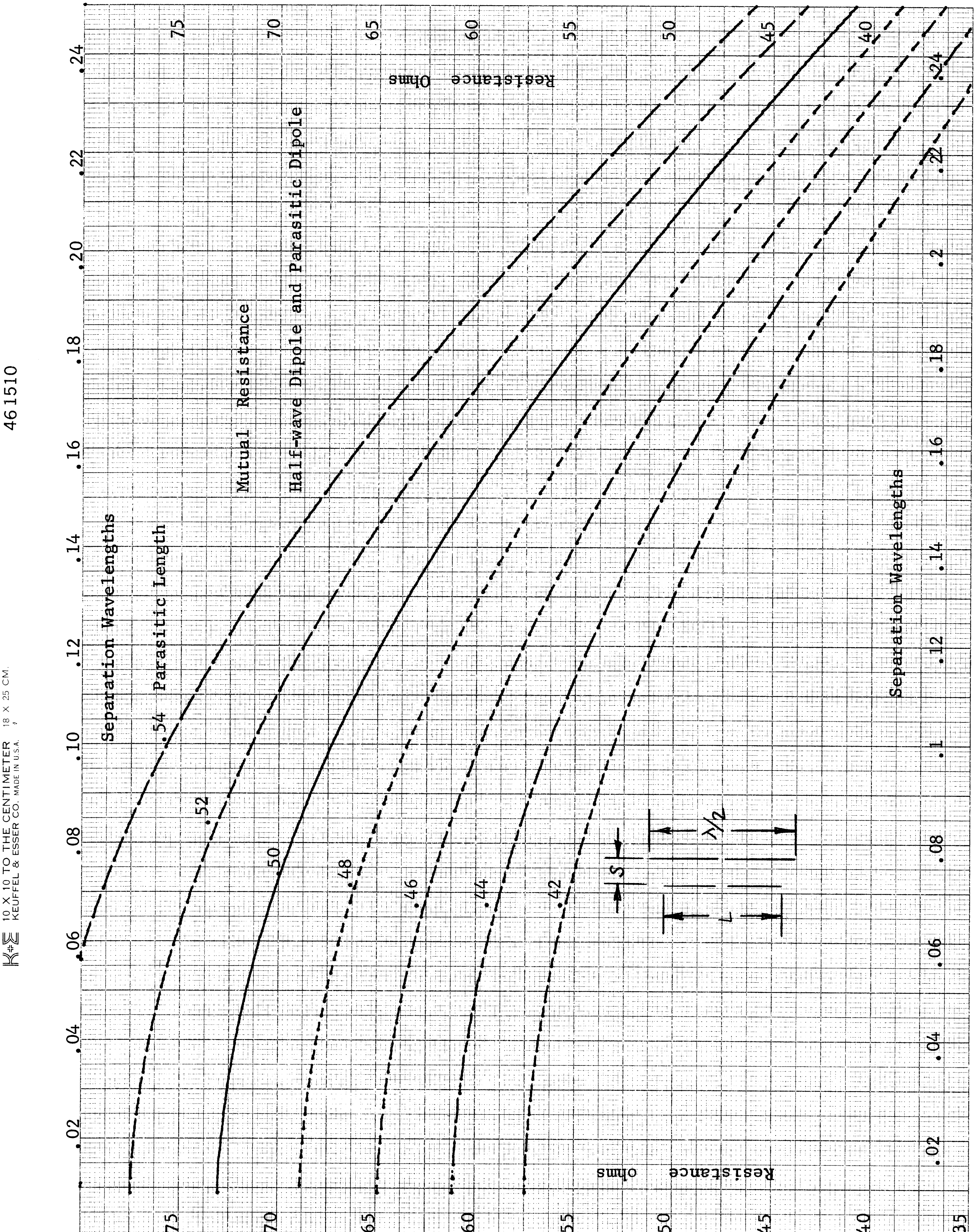
3. Hansen, R. C. (1966) Microwave Scanning Antennas, Vol. 2, Academic Press, New York, pp. 162f.

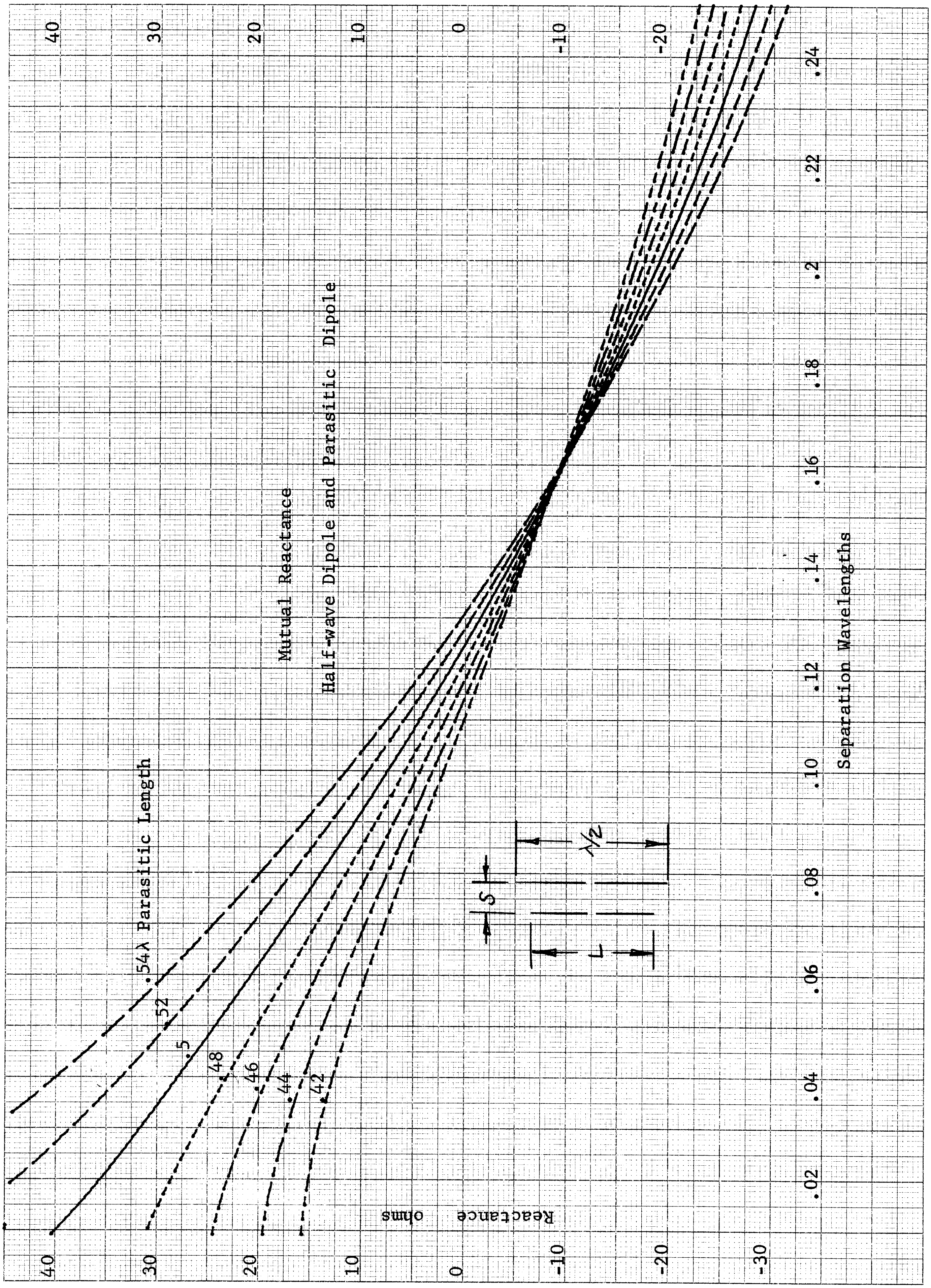
4. King, H. E., "Mutual Impedance of Unequal Length Antennas in Echelon", IRE Trans., Vol. AP-5, (July 1957), pp. 306-313.

5. Richmond, J. H., "Coupled Linear Antennas with Skew Orientation", IEEE Trans., Vol. AP-18, (September 1970), pp. 694-696.


Mutual Impedance
Broadside Half-wave Dipoles
Separation Wavelengths
Ohms
R
X
S
λ/2
0.5
1
1.5
2
80
60
40
20
0
-20
-40
-60


Mutual Impedance
Collinear Half-wave Dipoles
S
λ/2
Separation Wavelengths
Ohms
R
X
1
1.5
2
2.5
24
22
20
18
16
14
12
10
8
6
4
2
0
-2
-4
-6
-8


Mutual Resistance
Half-wave Dipole and Parasitic Dipole
Separation Wavelengths
Parasitic Length
Resistance Ohms
54
52
50
48
46
44
42
λ/2
S
L
46 1510
10 X 10 TO THE CENTIMETER 18 X 25 CM.
KEUFFEL & ESSER CO. MADE IN U.S.A.


Mutual Reactance
Half-wave Dipole and Parasitic Dipole
.54λ Parasitic Length
.52
.5
.48
.46
.44
.42
Separation Wavelengths
Reactance ohms
λ/2
S
L
40
30
20
10
0
-10
-20
-30
.02
.04
.06
.08
.10
.12
.14
.16
.18
.2
.22
.24

Slot Mutual Impedance

We can find the mutual impedance between two slots in a metal sheet if they are independently fed by using the Babinets - Booker relation given on page 182. The impedance is found by using the Carter results and the relation:

$$Y_{12(\text{Slot})} = \frac{4\, Z_{12(\text{Dipole})}}{\eta^2}$$

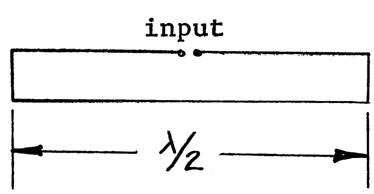
This can also be used to find the self impedance of the slot in the same manner as the radius was used to find the self impedance of a dipole. In this case we use the width of the slot as the distance. If the slots only radiate on one side of the ground plane, then from page 186 the admittance is only one-half that of the slot which radiates on both sides.

$$Y_{12(\text{Slot})} = \frac{2\, Z_{12(\text{Dipole})}}{\eta^2}$$

We can use circuit relationships to find properties of some antennas by using the mutual impedance concept.

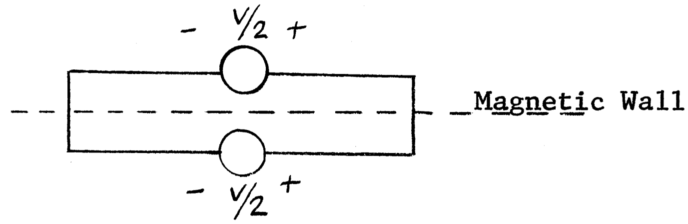
FOLDED DIPOLE

The folded dipole has the following configuration.

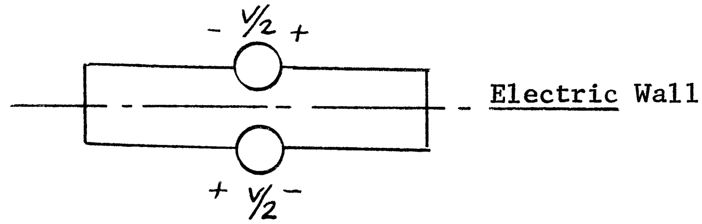


It is assumed that the two linear elements are tightly coupled. We will analyze this with even and odd mode analysis. The feed can be considered a combination of an even and odd mode feed. The input current is the sum of the two feed input currents.

Even Mode



Odd Mode



In the even mode there will be a magnetic wall between the two linear elements as shown in the figure above. As discussed on page 142, the magnetic wall separates the two elements into separate dipoles. The input impedance can be found from the mutual impedance matrix.

$$V_1 = Z_{11} I_1 + Z_{12} I_2 \qquad V_1 = V_2 = V/2$$

$$V_2 = Z_{12} I_1 + Z_{11} I_2 \qquad I_1 = I_2 = I_e$$

$$V/2 = (Z_{11} + Z_{12}) I_e \qquad I_e = \frac{V}{2(Z_{11} + Z_{12})}$$

The input voltages and currents are equal, which we can see from symmetry.

The odd mode feed will put an electric wall between the conductors. This wall shorts the elements and forms a transmission line between the electric wall and each conductor. The characteristic impedance of this line is one-half the impedance between the two lines. The odd mode is fed into two shorted stubs in series. The current into this mode is given by

$$I_o = \frac{V}{j Z_o \tan(\beta L/2)}$$

Where Z_o is the characteristic impedance between the two lines. This mode does not radiate because the equal and opposite currents together do not have a net far field. The input current is the sum of the two different mode currents.

$$I_{in} = \frac{V}{2(Z_{11} + Z_{12})} + \frac{V}{j Z_o \tan(\beta L/2)}$$

The input impedance is

$$Z_{in} = \frac{2(Z_{11} + Z_{12})}{1 - j\, 2(Z_{11} + Z_{12})/(Z_o \tan(\beta L/2))}$$

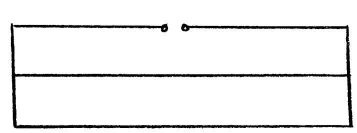
If the length, L, equals half wavelength, then the impedance becomes:

$$Z_{in} = 2(Z_{11} + Z_{12})$$

For closely coupled lines $Z_{12} = Z_{11}$ and the input impedance becomes:

$$Z_{in} = 4 Z_{11}$$

The folded dipole can be folded again to get a higher input impedance. The structure below can be analyzed by exciting the feed points on each line by



three fundamental modes. The radiating mode has an input voltage V/3 where

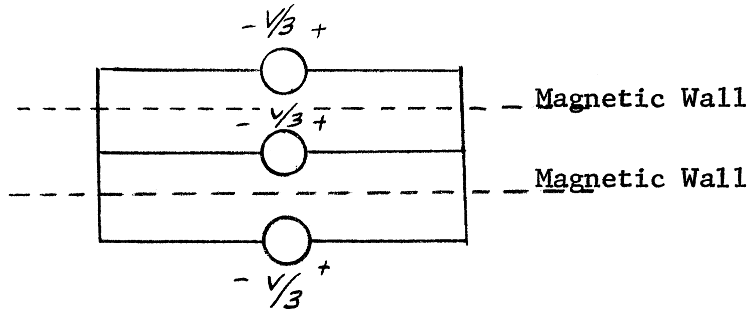
V is the total input voltage. The other two fundamental modes look like shorted transmission lines which at $\lambda/2$ are open circuits at the input. The input impedance is found from:

$$V/3 = (Z_{11} + Z_{12} + Z_{13})\, I$$

$$Z_{in} = 3(Z_{11} + Z_{12} + Z_{13})$$

Since the linear elements are closely coupled, $Z_{11} = Z_{12} = Z_{13}$ and $Z_{in} = 9\, Z_{11}$.

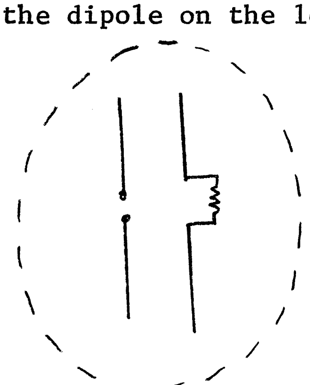
The radiating fundamental mode is as follows:



PARASITIC ELEMENTS

All the radiating structures studied so far have been fed directly by a transmission line. It is also possible to excite radiating elements by induction. These structures have currents induced on them by the fields from antenna elements with currents impressed upon them by sources. We can find the currents on these parasitic elements which will contribute to the far field pattern. The coupling between elements is a near field property, but our main concern is the effects on the far field pattern.

Consider two dipoles which are coupled broadside and close together in terms of wavelengths. When the dipole on the left is connected to a generator,



the dipole will set up a field around it due to the standing wave current on the dipole. The second dipole on the right is connected to a load. The fields of the first dipole will induce currents on the second dipole. We can find the direction of the currents by considering an area surrounding both elements as shown. By Lenz's law the currents induced on the element on the right will be in a direction to reduce the rate of change of magnetic flux through the area. The induced current is in the opposite direction of the currents on the fed dipole. The power for the currents comes out of the energy stored in the magnetic field set up by the currents of the driven dipole.

We can also solve the same problem by using the mutual impedance matrix without much physical insight.

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{12} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

Given a load on port 2, Z_2, we can solve for the input impedance of the two dipoles.

$$Z_{in} = \frac{V_1}{I_1} = Z_{11} - Z_{12}^2/(Z_{22} + Z_2)$$

The usual parastic element will have a short circuit on the input port to maximize the standing wave current on the element and to eliminate power dissipation in the load. In that case $Z_2 = 0$.

$$Z_{in} = Z_{11} - Z_{12}^2 / Z_{22}$$

From page 309 or 311 we see that the mutual impedance of two dipoles is very close to the self impedance of the dipole. As the two dipoles get closer together, the input impedance of the driven element approaches zero. Short circuiting the second dipole gives us a second equation relating the currents in the dipoles.

$$0 = Z_{12} I_1 + Z_{22} I_2$$

Solving we get

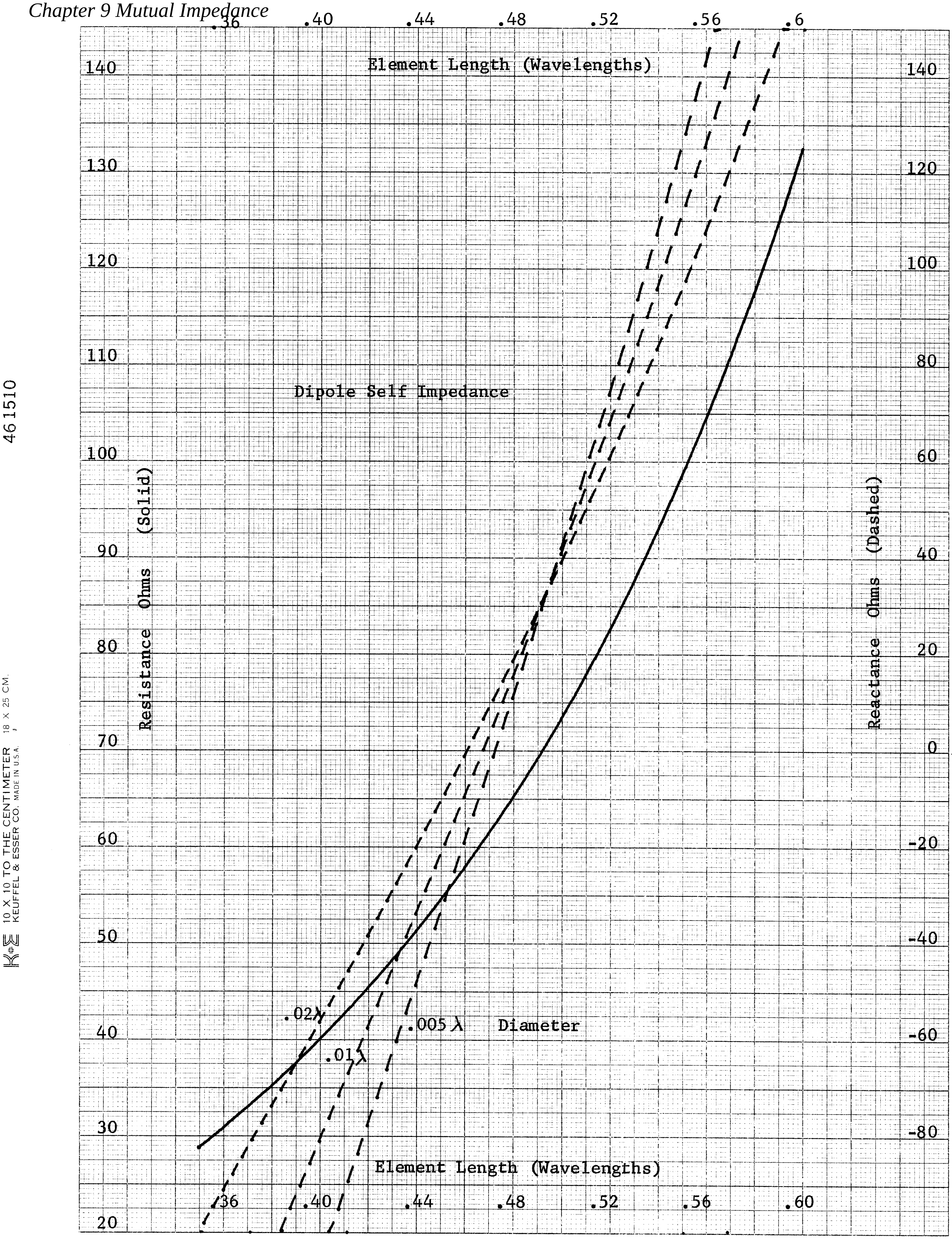
$$I_2 = - Z_{12} I_1 / Z_{22}$$

This is a mathematical statement of Lenz's (Faraday's) law. The current in the shorted dipole is in the opposite direction of the current in the driven element for closely coupled dipoles where $Z_{12} \approx Z_{22}$.

Once we have the standing wave current in the parastic element, we can solve for the far field pattern by using array techniques. There are two radiating elements. In order to find the current in the parastic element we must know the mutual coupling between the antenna and the parastic dipole. On pages 311 and 312 are plotted the mutual resistance and reactance of two closely spaced broadside coupled dipoles with the feed element $\lambda/2$ and the parastic element near $\lambda/2$. Finally on page 317 is a plot of the self impedance of an element for a range of lengths near a half wavelength.

Suppose we have a half-wave element and a parasitic element spaced 0.1 wavelengths with the second element 0.42 wavelengths long. The mutual impedance is given by the curves on pages 311 and 312 as:

$$Z_{12} = 53 + j\,2$$



.36
.40
.44
.48
.52
.56
.6
Element Length (Wavelengths)
140
130
120
110
100
90
80
70
60
50
40
30
20
140
120
100
80
60
40
20
0
-20
-40
-60
-80
Dipole Self Impedance
Resistance Ohms (Solid)
Reactance Ohms (Dashed)
.02λ
.01λ
.005 λ
Diameter
Element Length (Wavelengths)
.36
.40
.44
.48
.52
.56
.60
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The self impedance is given on the curve on page 317.

$$Z_{22} = 45.6 - j\,77$$

Using these, the current in the parasitic dipole is given as

$$I_2 = -Z_{12}\, I_1 / Z_{22} = -(53 + j2) I_1 / (45.6 - j77) = .59 I_1 \underline{/241^\circ}$$

The current in the parasitic dipole is 59% of the current in the feed dipole. The coupling between them is good. In circuits we are not use to such high coupling because there is a ground plane which will reduce the coupling to low levels. To achieve high coupling the parts of the circuit must be very close such as in a directional coupler.

Now we must solve for the pattern of these two elements. We can use the results of an array of two elements. Each dipole has the same pattern (or at least very close). The normalized far field electric field is given by

$$E = 1 + I_r\, e^{j(\beta d \cos\theta + \delta)}$$

$$\text{where} \quad I_r\, e^{j\delta} = \frac{I_2}{I_1}$$

for two elements spaced on the Z axis. Let us consider the pattern at $\theta = 0$ and $\theta = 180^\circ$.

$$\theta = 0 \qquad \cos\theta = 1$$

$$\theta = 180 \qquad \cos\theta = -1$$

$$E(0) = 1 + I_r\, e^{j(\beta d + \delta)}$$

$$E(180) = 1 + I_r\, e^{j(\delta - \beta d)}$$

We want to compare the pattern magnitude at $\theta = 0$ with the pattern at $\theta = 180^\circ$. Expand the exponential function with the complex argument by Euler's identity.

$$E(0) = 1 + I_r \cos(\delta + \beta d) + j\, I_r \sin(\delta + \beta d)$$

$$E(180) = 1 + I_r \cos(\delta - \beta d) + j\, I_r \sin(\delta - \beta d)$$

The magnitude of the pattern is

$$|E(0)|^2 = (1 + I_r \cos(\delta + \beta d))^2 + I_r^2 \sin^2(\delta + \beta d)$$

$$|E(180)|^2 = (1 + I_r \cos(\delta - \beta d))^2 + I_r^2 \sin^2(\delta - \beta d)$$

Expanding these and using the trigonometic identity: $\cos^2 A + \sin^2 A = 1$

we get the result

$$|E(0)|^2 = 1 + I_r^2 + 2\, I_r \operatorname{Cos}(\delta + \beta d)$$

$$|E(180)|^2 = 1 + I_r^2 + 2\, I_r \operatorname{Cos}(\delta - \beta d)$$

The difference between the two responses is

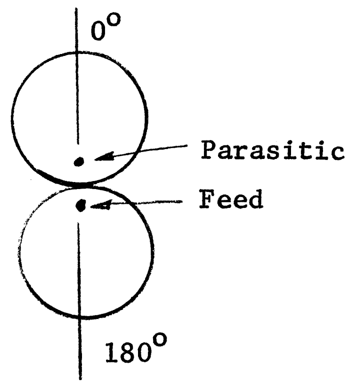
$$|E(0)|^2 - |E(180)|^2 = 2\, I_r\, (\operatorname{Cos}(\delta + \beta d) - \operatorname{Cos}(\delta - \beta d))$$

We can use another trigonometric identity to get

$$\Delta E^2 = |E(0)|^2 - |E(180)|^2 = -2\, I_r \operatorname{Sin} \delta \operatorname{Sin} \beta d$$

$$\Delta E^2 = 0 \quad \text{if} \quad \delta = 180^o$$

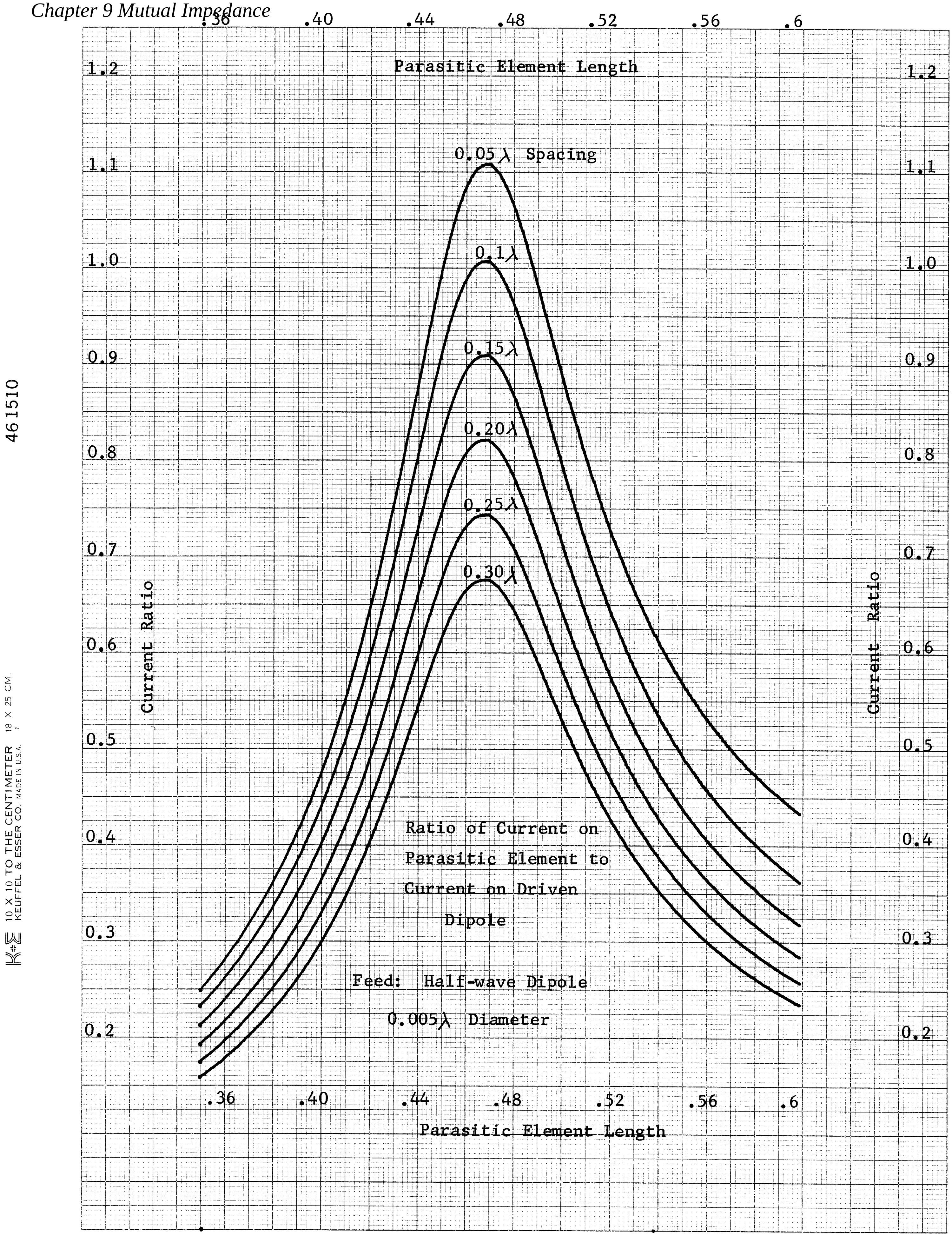
In this case the beams in the direction of $\theta = 0$ and $\theta = 180^o$ are equal and the pattern is approximately given as:



The two currents are 180° out of phase so there will be nulls at $\theta = 90^o$ if the currents are equal and a dip if not equal. Regardless of the level of the two currents, the pattern will be equal at $\theta = 0$ and $\theta = 180$ if the phase between the currents is 180°.

If $180^o < \delta < 360^o$, then the parasitic dipole will act as a director and the pattern at 0° will be higher than the pattern at 180°. Similiarly, if $0 < \delta < 180^o$, then the parasitic dipole will act as a reflector and the pattern will be higher at 180°. In the example above the angle is 241° which means the parasitic dipole is a director.

On page 320 is a plot of the relative phase angle of the current on the parasitic dipole relative to the current on the driven dipole. On this plot we can see the region where the parasitic dipole is a director and a reflector. The same curve for a feed dipole length of 0.47 wavelengths was drawn and is almost an overlay of the curve on page 320. The magnitude of the relative current is plotted on page 321. Notice that the parasitic element for a spacing of 0.05 wavelengths is a director for lengths less than .486 wavelengths, but for a spacing of 0.25 the element is a director for lengths less than .447 wavelengths. It is interesting that the current on the parasitic dipole is higher than the feed element for close spacings.



.36
.40
.44
.48
.52
.56
.6
Parasitic Element Length
1.2
1.1
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.05λ Spacing
0.1λ
0.15λ
0.20λ
0.25λ
0.30λ
Current Ratio
Ratio of Current on
Parasitic Element to
Current on Driven
Dipole
Feed: Half-wave Dipole
0.005λ Diameter
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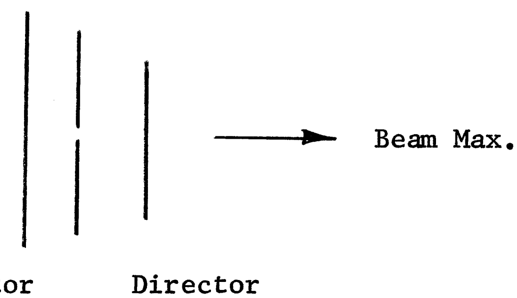
YAGI - UDA

This type of antenna was first reported in 1928 by Yagi[1]. It uses parasitic elements as reflectors and directors around the feed element to direct the energy into an endfire pattern. The antennas with a single parasitic element are included in the general Yagi type antenna which we have considered above. The next simpliest Yagi antenna has one director and one reflector besides the feed element.



The central feed dipole must be fed like any other dipole with a balanced transmission line to prevent squint in the pattern from currents induced on the feed lines.

Almost all Yagi-Uda antennas have been designed experimentally and have moderately narrow bandwidths. Depending on the application, the antenna may be designed for maximum gain, maximum front to back ratio, or even flat response over a band of frequenies. It would seem that the maximum gain would occur for the best front to back ratio, but we will see that this is not the case. We will consider some designs in the literature but first we must discuss the analysis.

The Yagi-Uda antenna works by coupling energy from the feed dipole to a series of parasitic elements which re-radiate the energy due to standing wave currents induced on them. Once we have found the currents on all the elements, we can use array theory to calculate the radiation pattern. The antenna is described by a mutual impedance matrix.

$$\begin{bmatrix} V_1 \\ V_2 \\ \cdot \\ \cdot \\ \cdot \\ V_N \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} & \cdots & Z_{1N} \\ Z_{12} & Z_{22} & \cdots & Z_{2N} \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot \\ Z_{1N} & Z_{2N} & \cdots & Z_{NN} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ \cdot \\ \cdot \\ \cdot \\ I_N \end{bmatrix}$$

The elements of the mutual impedance matrix are found by standard formulas for mutual impedance assuming the antennas interact on a one to one basis. That is, in pairs of antennas without the other ones present. It has been found experimentally that adding more than one reflector has little effect

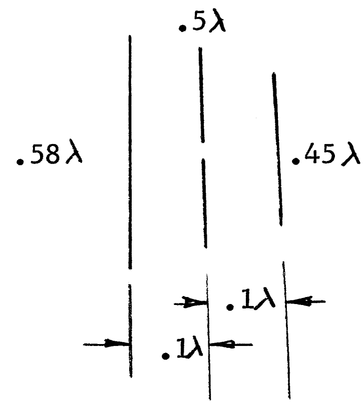
1. H. Yagi, "Beam Transmission of Ulta Short Waves", Proc. IRE 16 (June 1928)

on the pattern so we can assume 1 is the reflector dipole and 2 is the feed dipole. The input voltage is a vector with one one non-zero entry and we can solve the matrix equation for the currents.

$$\begin{bmatrix} I_1 \\ I_2 \\ \cdot \\ \cdot \\ \cdot \\ I_N \end{bmatrix} = Z^{-1} \begin{bmatrix} 0 \\ 1 \\ 0 \\ \cdot \\ \cdot \\ 0 \end{bmatrix}$$

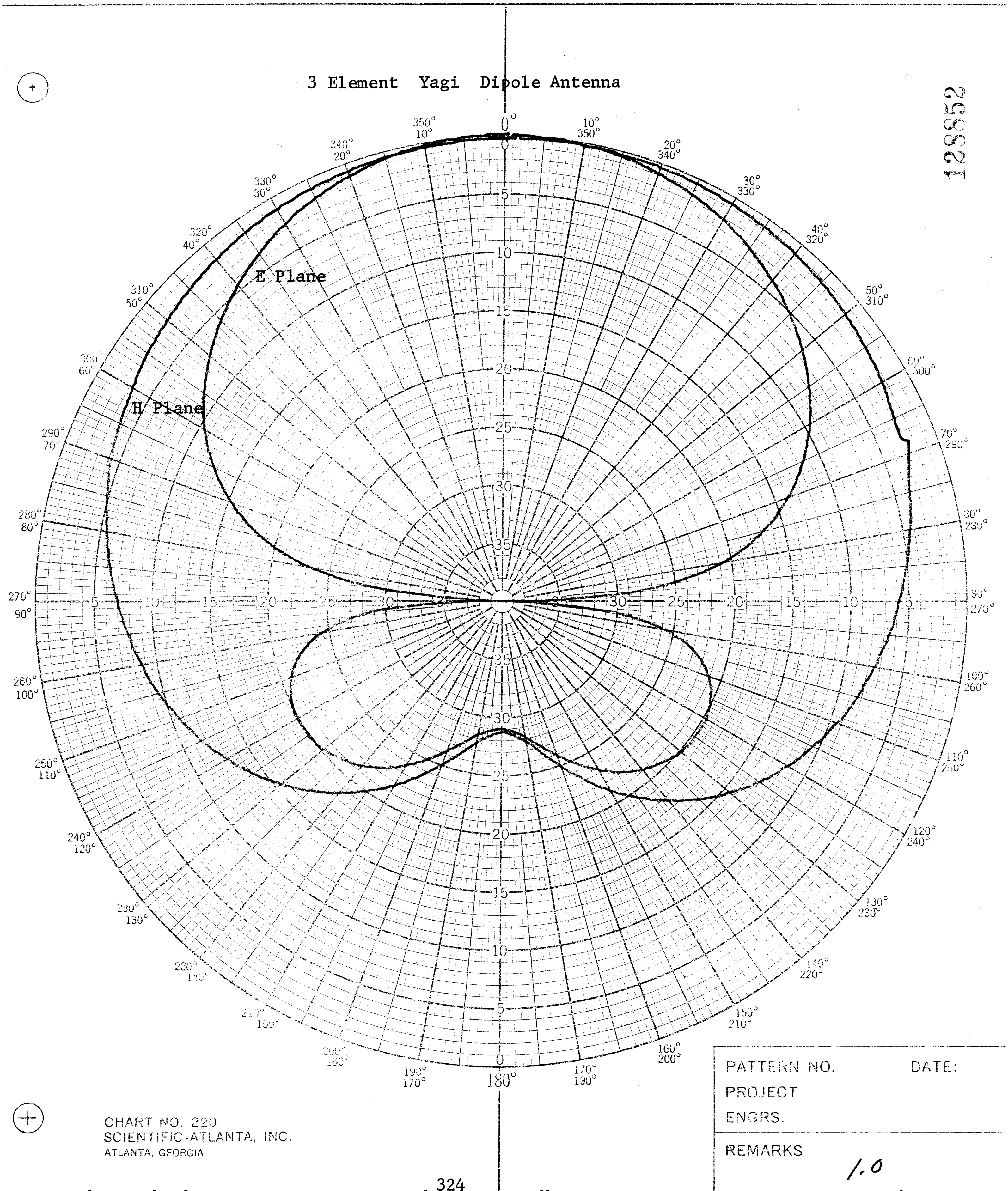
Where Z^{-1} is the matrix inverse of the mutual impedance matrix. This method is one of the special cases of the general method of moments. We saw that with a single parasitic element the input impedance tends toward zero as the elements are brought closer together. In general this holds true for N element Yagi antennas, but not always. One solution to this problem is to use a folded dipole as the feed dipole and raise the input impedance by a factor of 4. The folded dipole will give approximately the same results as the single dipole feed.

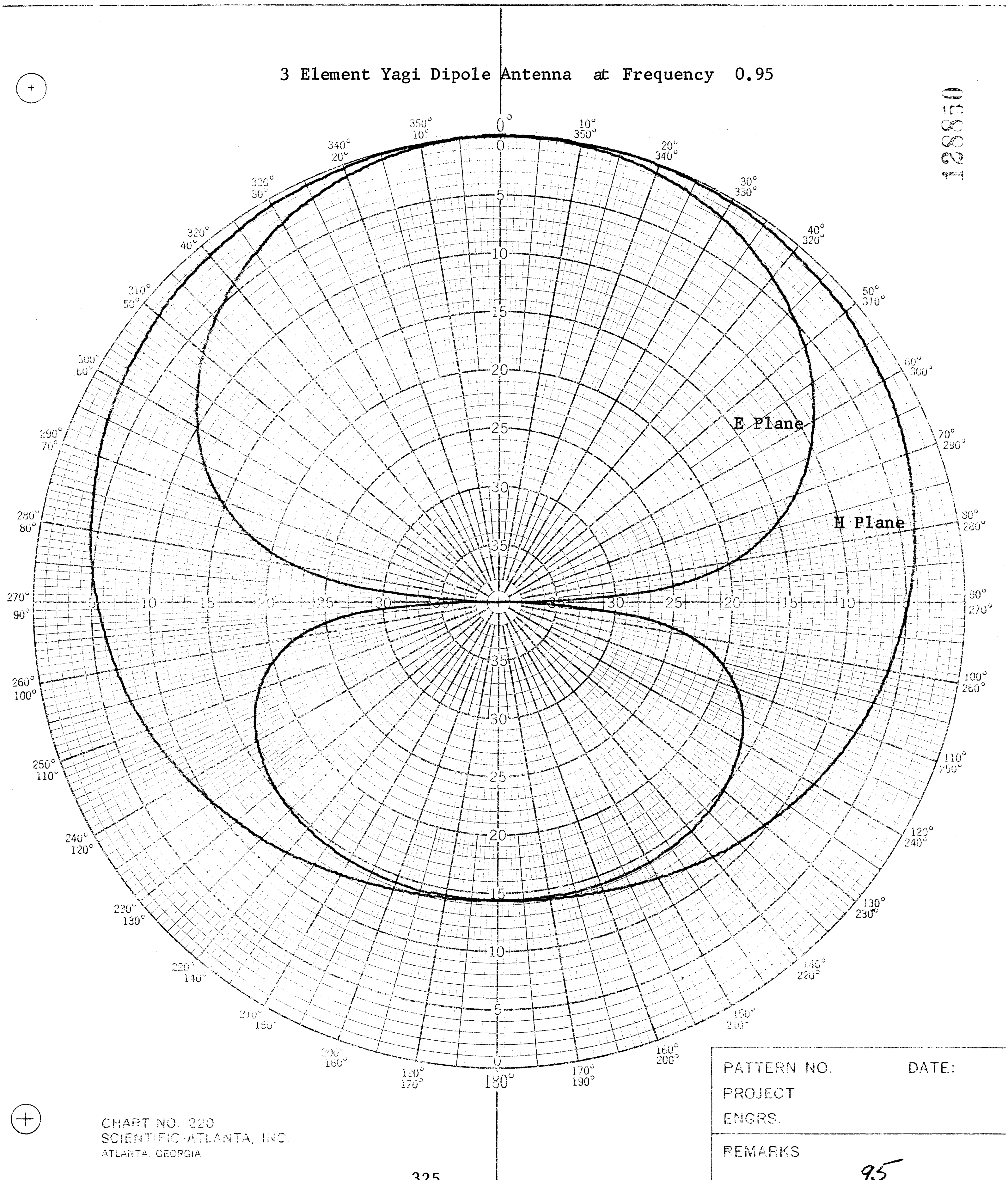
The following 3 element Yagi-Uda dipole antenna is given by Kraus.

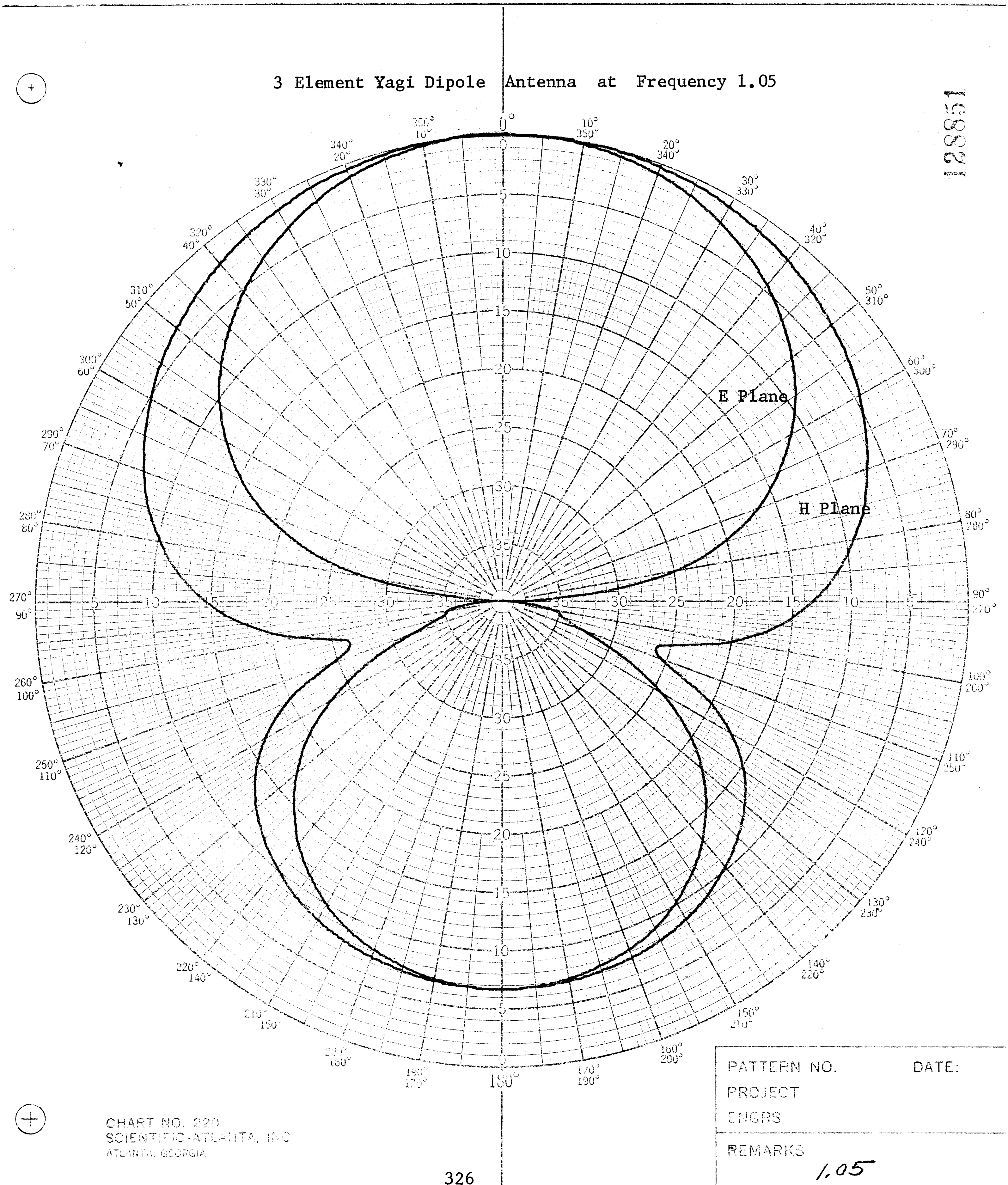


The center frequency pattern of this antenna is plotted on page 324. This antenna was designed to give a good front to back ratio (29 dB). The E plane pattern shows a typical null at $\theta = 90^{o}$ due to the dipole element pattern. Any array of dipoles will have this null. The H plane pattern is very broad; only coming in at the point of the back lobe reduction. This is more or less the typical response of the H plane. The antenna has a directivity of 6.8 dB at the center frequency.

When the frequency moves off center frequency, the pattern responses start degrading. On pages 325 and 326 are patterns of the antenna at plus and minus 5% off the center frequency. Note that the front to back ratio has been reduced but in the case of a normalized frequency of 1.05, the directivity has increased to 7.8 dB. Even though the back lobe has increased, the H plane pattern beamwidth has decreased for a net gain.


3 Element Yagi Dipole Antenna
E Plane
H Plane
12852
CHART NO. 220
SCIENTIFIC-ATLANTA, INC.
ATLANTA, GEORGIA
PATTERN NO.
DATE:
PROJECT
ENGRS.
REMARKS
1.0


3 Element Yagi Dipole Antenna at Frequency 0.95
E Plane
H Plane
CHART NO. 220
SCIENTIFIC-ATLANTA, INC.
ATLANTA, GEORGIA
PATTERN NO.
DATE:
PROJECT
ENGRS.
REMARKS
.95


3 Element Yagi Dipole Antenna at Frequency 1.05
E Plane
H Plane
128851
PATTERN NO.
DATE:
PROJECT
ENGRS
REMARKS
1.05
CHART NO. 220
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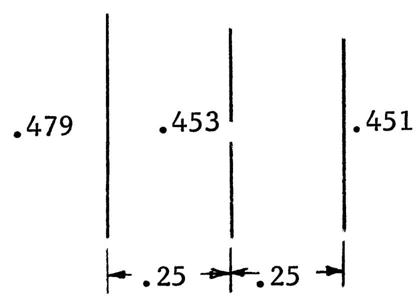
A curve of the frequency response has been plotted on page 328. The first curve is the gain if the antenna is matched at each frequency to the transmission line; this is the directivity. The directivity response is fairly flat below the center frequency but falls off rapidly above 1.05 normalized frequency. The second curve is the gain to a 50 ohm transmission line source. Because the match of the antenna degrades rapidly, the gain falls off even more rapidly than the directivity above the center frequency. The last curve is the front to back ratio (F/B). The antenna was designed to maximize this ratio and the response falls off rapidly when the frequency is changed. The scale on the right is used with this curve. Notice that the F/B becomes negative for frequencies above 1.08. This means that the antenna radiates in the backfire direction (toward the reflector).

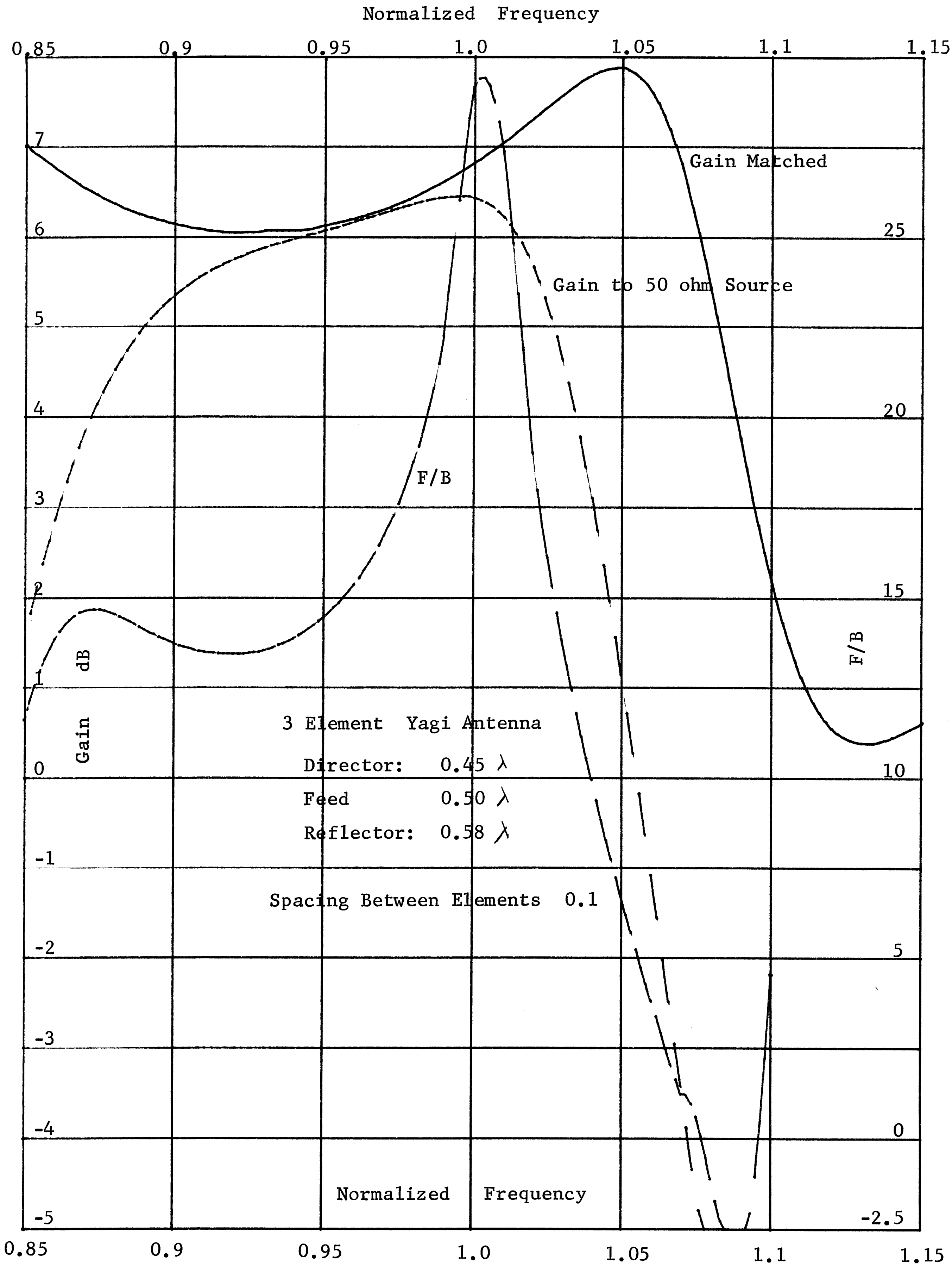
The input impedance of this antenna is plotted on a Smith chart on page 329. The reflection coefficient angle rotates in the negative direction with increasing frequency which is a consequence of Foster's reactance theorem. This plot shows that a reasonable input match is only possible over a limited band of frequencies.

The second example is given by Stutzman and Thiele.



This antenna has been designed to give maximum gain at the center frequency for quarter-wave spacings. The calculated gain is 9.5 dB. The pattern of this antenna at center frequency is given on page 330. Notice that the front to back ratio is only 6.6 dB. The high gain is achieved by narrowing the H plane pattern and the E plane pattern comparied to the design given by Kraus for good front to back ratio. The input impedance is about 19 ohms at the center frequency. The gain and F/B frequency response is plotted on page 331. The gain is maximized at the center frequency. The antenna never achieves a good F/B and like the first example, negative F/B indicates an antenna which radiates in the direction of the reflector.

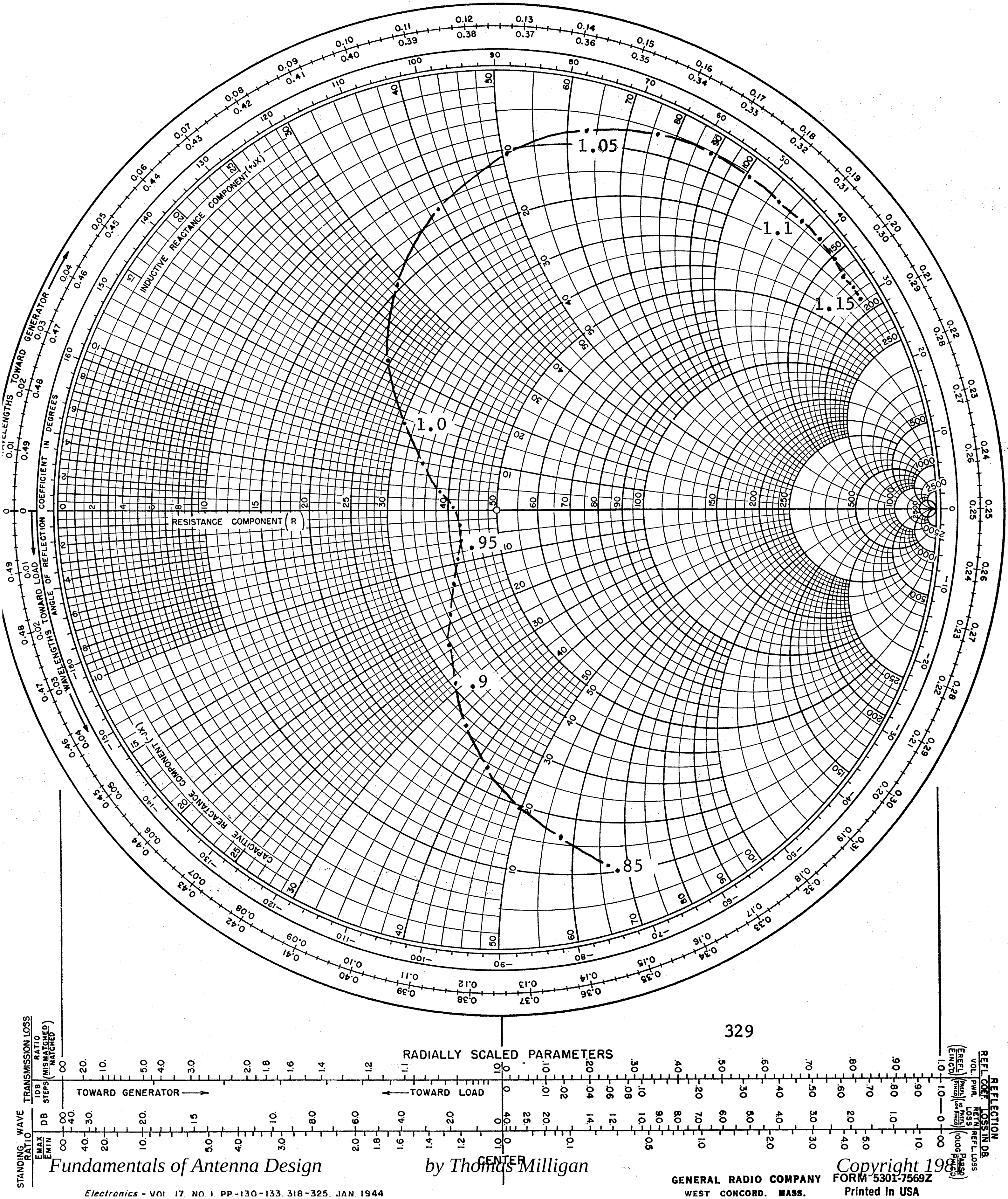
The design for the third example is also from Stutzman and Thiele. A seven element Yagi with equal spacings and equal directors is given. All the spacings are 0.25 wavelengths and all directors are 0.434 wavelengths. The feed is 0.454 wavelengths and the reflector is 0.477 wavelengths. The frequency response curves are plotted on page 332. This antenna is practically matched at the center frequency so that the gain to a 50 ohm source almost equals the directivity. The gain at center frequency is about 12 dB. This gain points out the limitations of Yagi-Uda antennas. The gain has not increased very much for a large increase in the number of elements. Because the added elements are further away from the feed element

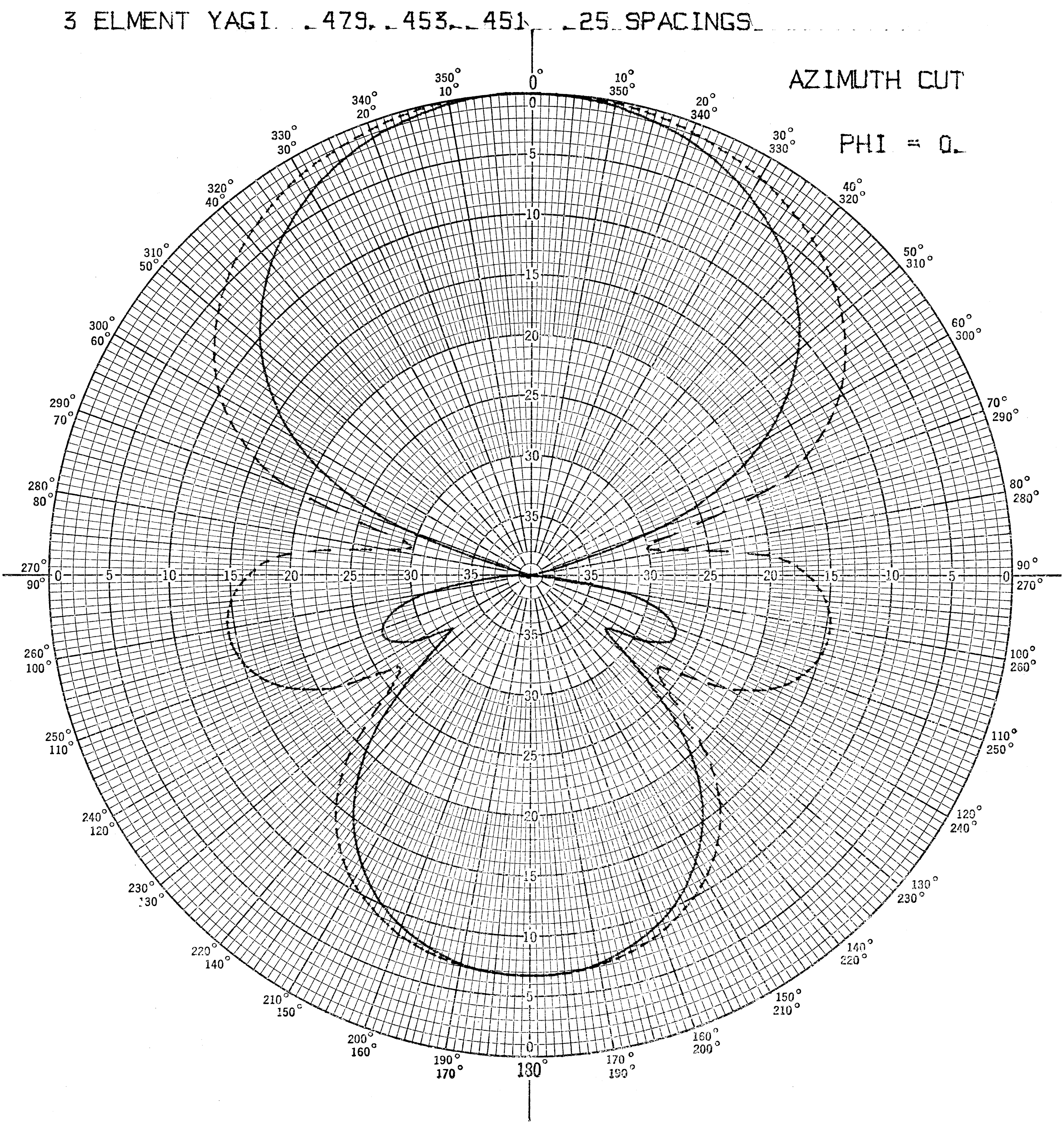

Normalized Frequency
0.85
0.9
0.95
1.0
1.05
1.1
1.15
7
6
5
4
3
2
1
0
-1
-2
-3
-4
-5
25
20
15
10
5
0
-2.5
Gain Matched
Gain to 50 ohm Source
F/B
Gain dB
F/B
3 Element Yagi Antenna
Director: 0.45 λ
Feed 0.50 λ
Reflector: 0.58 λ
Spacing Between Elements 0.1
Normalized Frequency

TITLE 3 Element Yagi Antenna Input Impedance

DATE

IMPEDANCE COORDINATES—50-OHM CHARACTERISTIC IMPEDANCE





Polar Chart No. 127D
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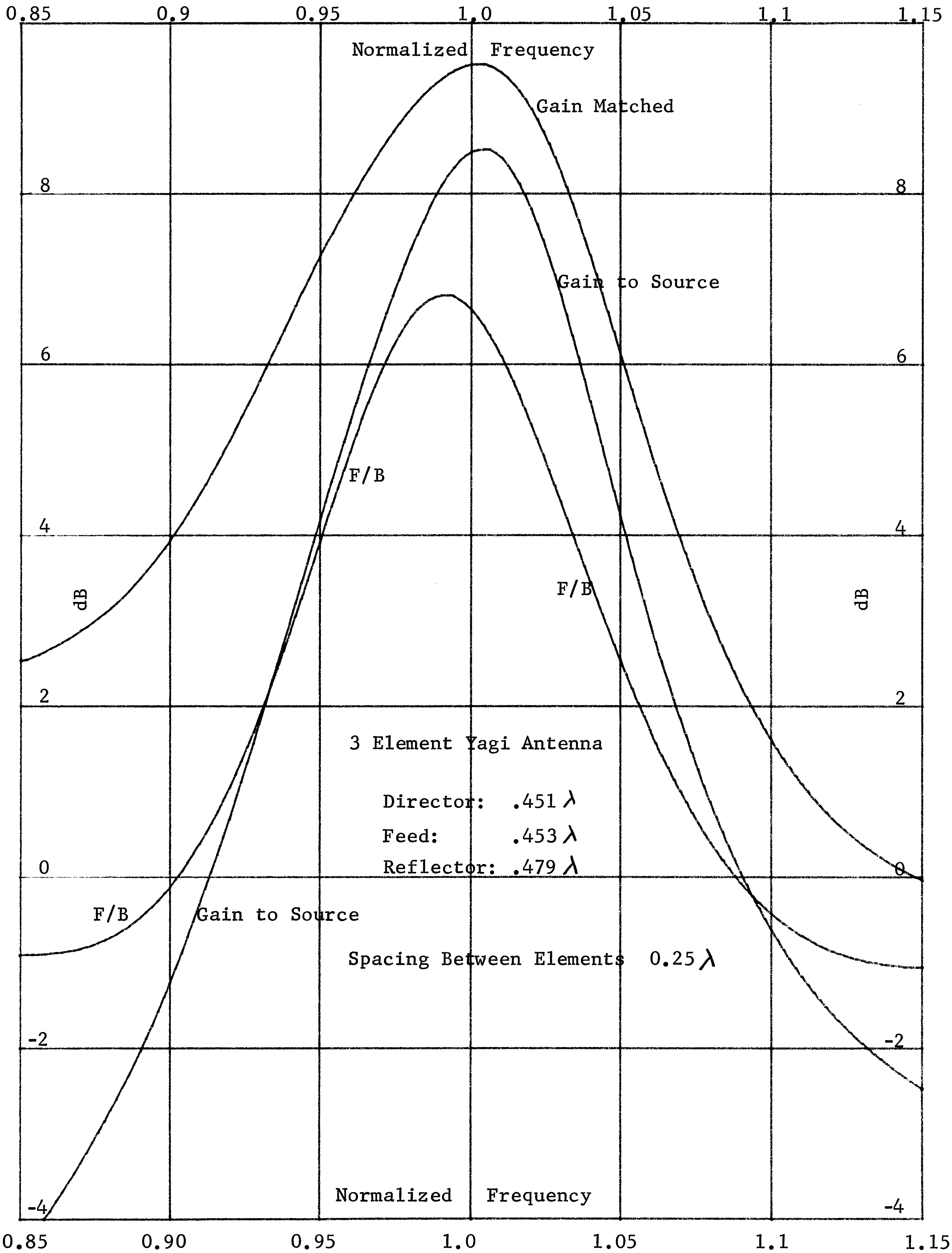
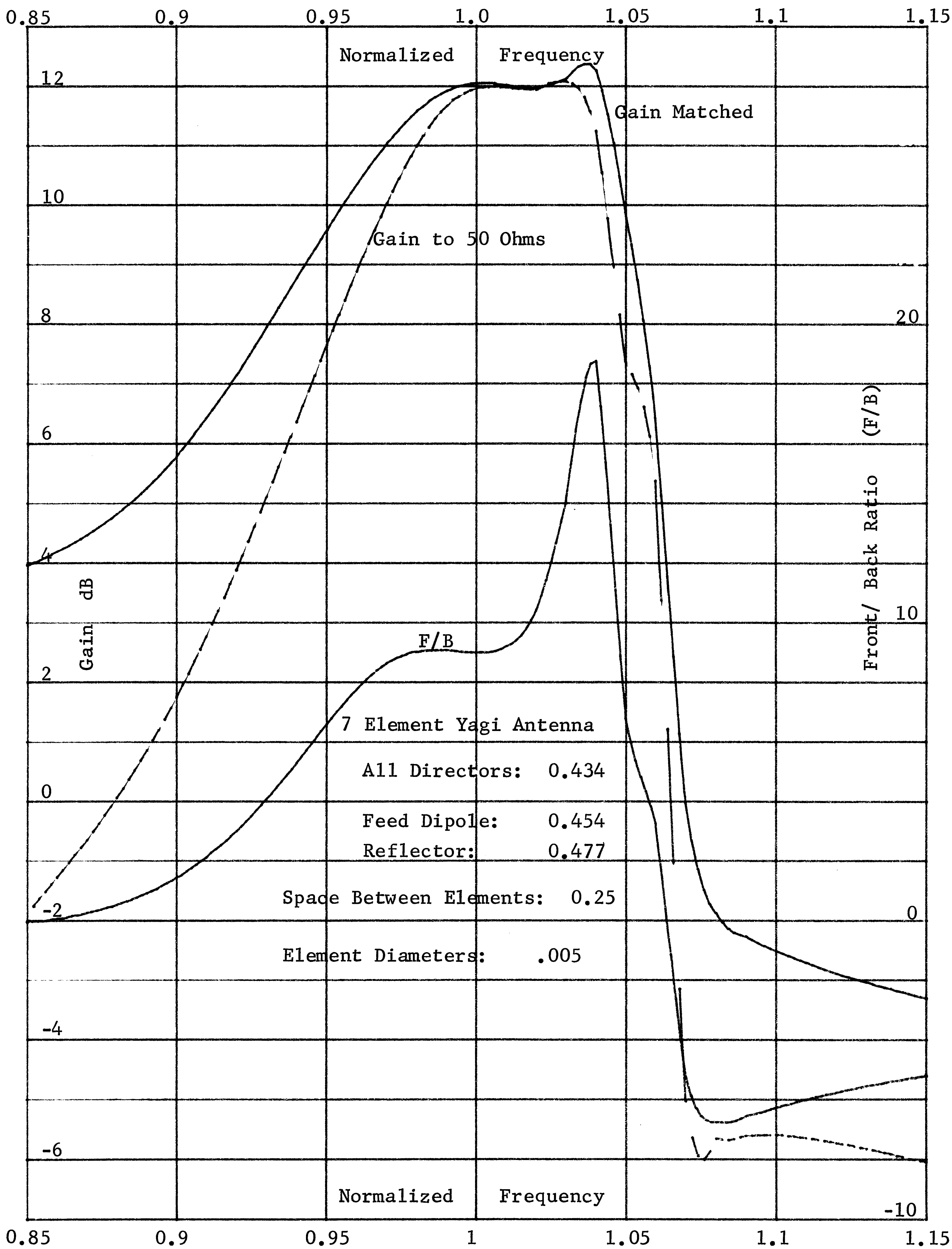


0.85
0.9
0.95
1.0
1.05
1.1
1.15
Normalized Frequency
Gain Matched
8
Gain to Source
6
F/B
4
dB
F/B
2
3 Element Yagi Antenna
Director: .451 λ
Feed: .453 λ
Reflector: .479 λ
0
F/B
Gain to Source
Spacing Between Elements 0.25 λ
-2
-4
Normalized Frequency
0.85
0.90
0.95
1.0
1.05
1.1
1.15



0.85
0.9
0.95
1.0
1.05
1.1
1.15
Normalized
Frequency
Gain Matched
Gain to 50 Ohms
F/B
Gain dB
Front/ Back Ratio (F/B)
12
10
8
6
4
2
0
-2
-4
-6
20
10
0
-10
7 Element Yagi Antenna
All Directors: 0.434
Feed Dipole: 0.454
Reflector: 0.477
Space Between Elements: 0.25
Element Diameters: .005
Normalized
Frequency
0.85
0.9
0.95
1
1.05
1.1
1.15

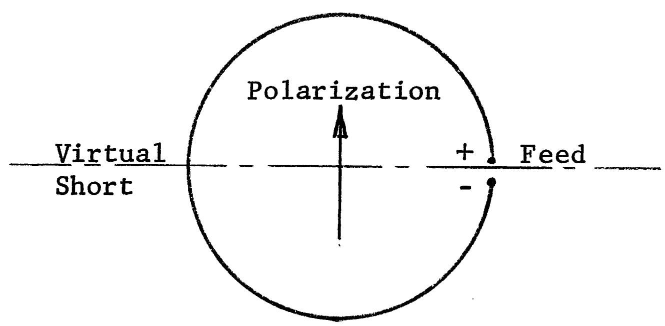
and the induced current falls off when compared to the reflector and first couple of directors. After about 20 elements, there is no significant increase in the gain of the antenna for added elements.

Some attempts have been made to use optimization routines to design these antennas. There are problems with these methods. First there are both element lengths and spacings. The dimensions of the search space becomes very large because there is a dimension for each variable. A three element Yagi requires a search in 5th order space. The second problem is that there are local maximums and minimums which the optimization routines get caught in and do not escape. The third problem is that the two closest elements to the feed are the most significant elements and the routines spend all the time changing those without changing the further out elements. It still appears that an experimental approach is the best design method.

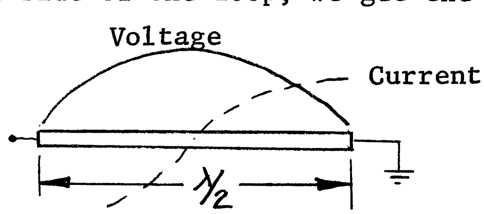
RESONANT LOOP

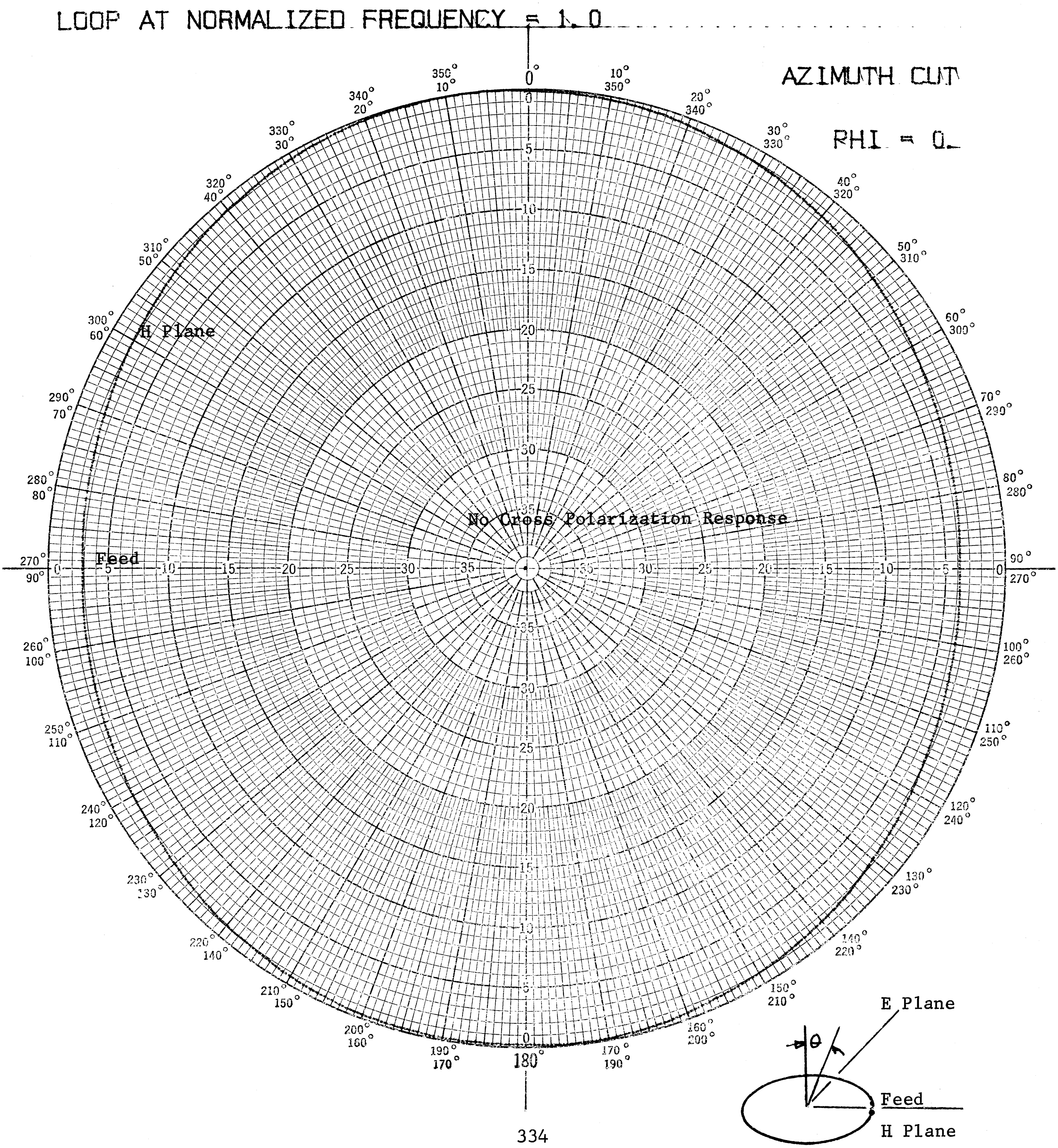
A loop one wavelength in circumference will resonate and have a maximum pattern response normal to the plane of the loop. The patterns of the loop are given on pages 334 and 335 for a loop in the X-Y plane. The H plane pattern has a slightly higher response at the side of the feed than the side opposite. The H plane is through the feed point. The antenna is linearly polarized with the electric field aligned with the voltage across the feed point. The H plane pattern has no cross polarization response, but the E plane has a cross polarization response off boresight. This cross polarization response increases for frequencies off resonance.

We can understand the loop by the following argument. When the loop is fed by a balanced transmission line feed, there is a virtual short circuit on the loop directly opposite the feed. If we think of the loop as a pair of transmission lines, the pair is in the odd mode. But we can equally

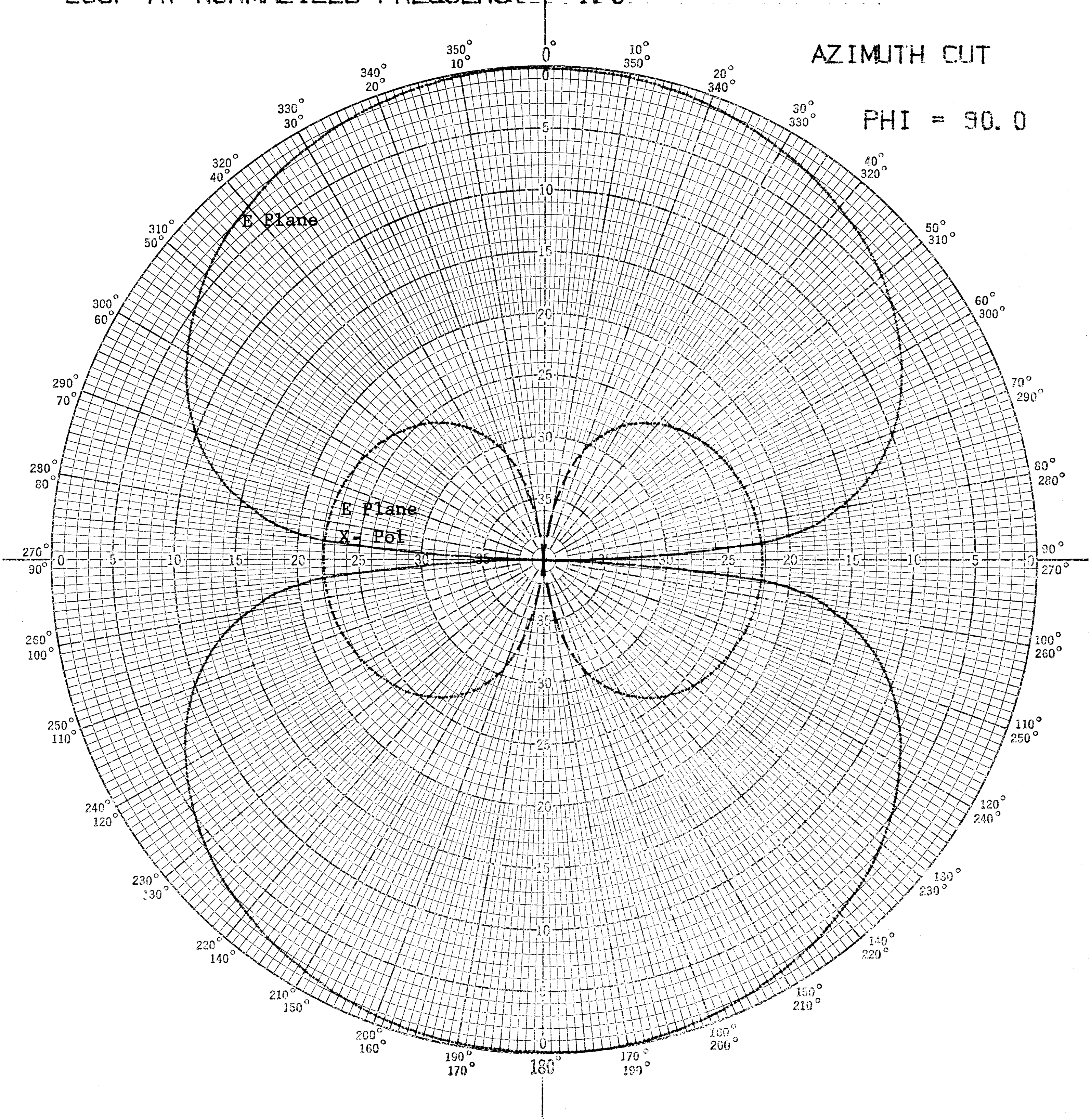


look on the loop as a single transmission line, like we did the dipole, where the two conductors are shorted together at the end. Shorting a transmission line will give a current maximum at that point and of course a voltage minimum. We will set up a sinusoidal standing wave on the loop. Considering only one side of the loop, we get the following standing wave.




LOOP AT NORMALIZED FREQUENCY = 1.0
AZIMUTH CUT
PHI = 0
H Plane
No Cross Polarization Response
Feed
E Plane
θ
Feed
H Plane
0°
180°
90°
270°

LOOP AT NORMALIZED FREQUENCY - 1.0



Polar Chart No. 127D
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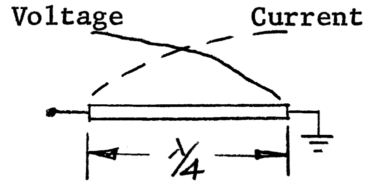
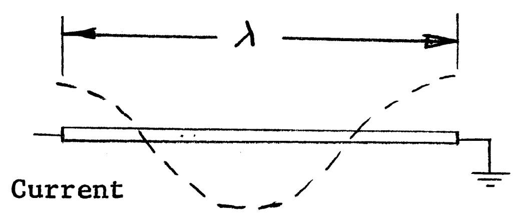
This is similiar to the dipole. We get a reasonable input impedance when there is a current maximum at the input and very high impedance when it is a minimum. For a loop a half-wavelength in circumference we get a current minimum at the input and the input impedance is very high.



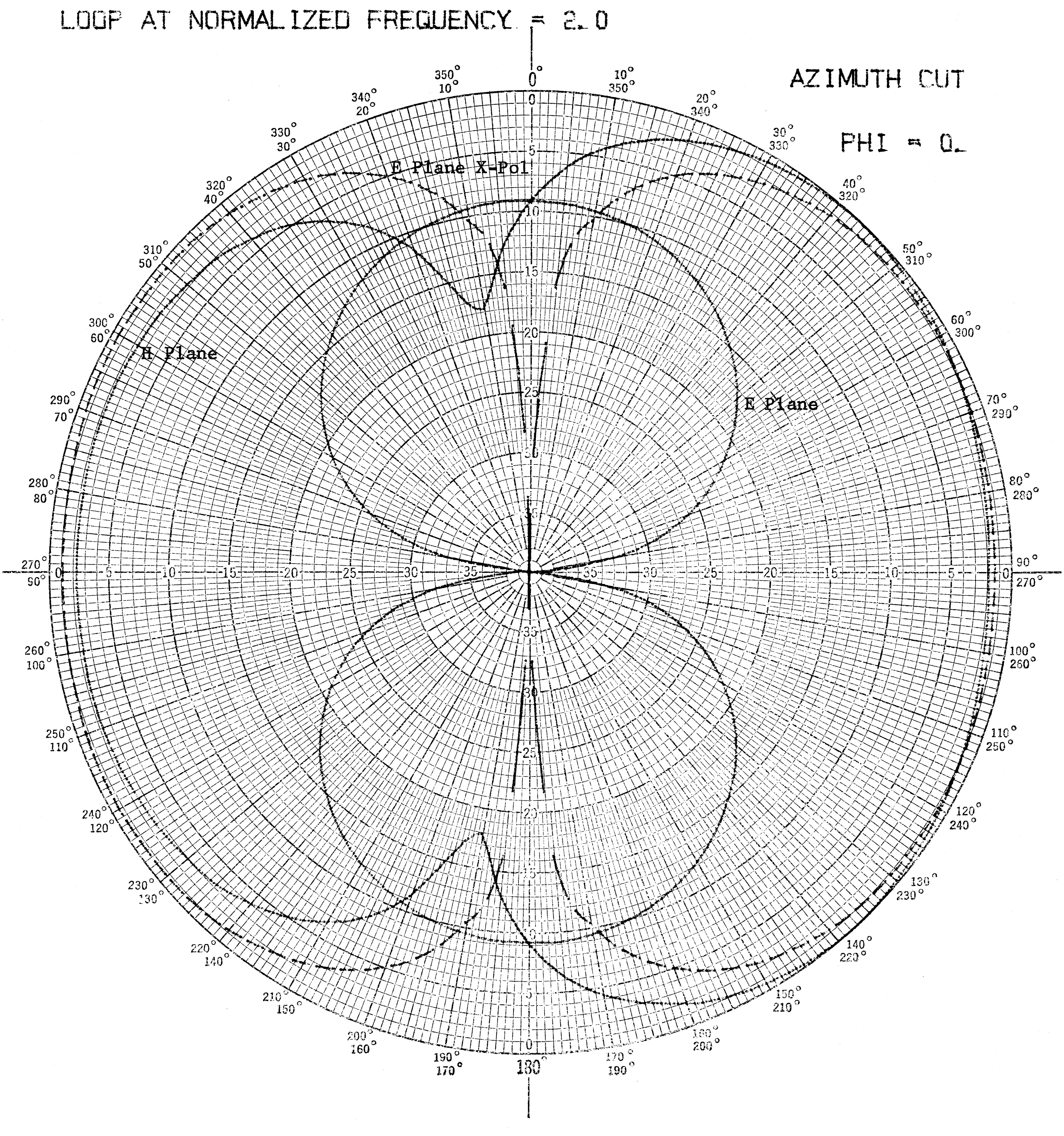
This is true for any odd multiple of a half wavelength circumference loop. The loop will have a reasonable input impedance again near two wavelengths circumference length, but the pattern has broken up. The cross polarization response in the E plane has increased to the level of the H plane response and nulls are appearing at boresight. The pattern is plotted on page 337. This is starting to approach the pattern from a uniform current loop which has only a ϕ component and a null on boresight. But this is not the case. The pattern null is caused by a standing wave current analogous to the one wavelength offset fed dipole which has current asymmetry on the dipole.



On page 338 is a plot of the input impedance of the loop versus normalized frequency near resonance where the loop has a circumference of one wavelength at F = 1. The loop has zero reactance for a circumference a little longer than one wavelength. We need to compare the reactance slope of the loop at resonance to the reactance slope of the dipole near resonance found on page 317. The reactance slope of the loop is 2.5 times that of the dipole. This means that the bandwidth of the loop is 2.5 times less than the dipole, since bandwidth is proportional to the reciprocal of the reactance slope.

The directivity of the loop is about 3.7 dB for a resonated loop. This can be comparied to the one wavelength dipole whose gain is 3.8 dB.

It is not necessary that the loop be circular. One page 339 is the pattern of a square loop whose perimeter equals one wavelength. This pattern is close to the circular loop except that the cross polarization in the E plane is higher. The feed is located in the center of one of the straight sections. The square loop may also be fed at one of the corners as shown in the pattern on page 340. In both cases it appears from the input impedance that the square loop should be 1.11 wavelengths long to resonate out the reactance, a little longer than the circular loop. On page 341 is the pattern of a triangular loop, one wavelength in perimeter. It too has approximately the same pattern as the circular loop. The triangular loop must be made 1.125 wavelengths long to resonate.


LOOP AT NORMALIZED FREQUENCY = 2.0
AZIMUTH CUT
PHI = 0.
E Plane X-Pol
H Plane
E Plane
Polar Chart No. 127D
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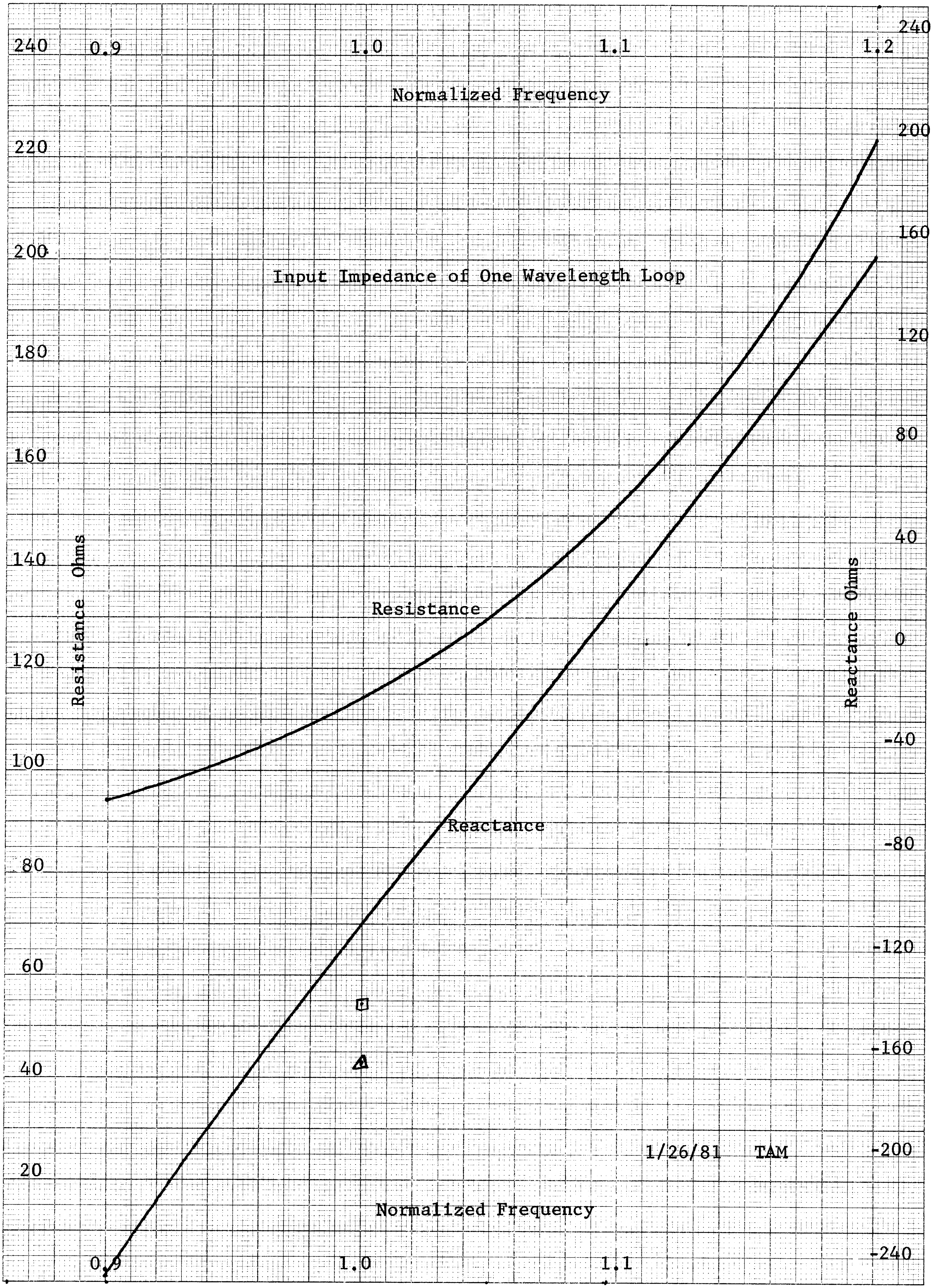
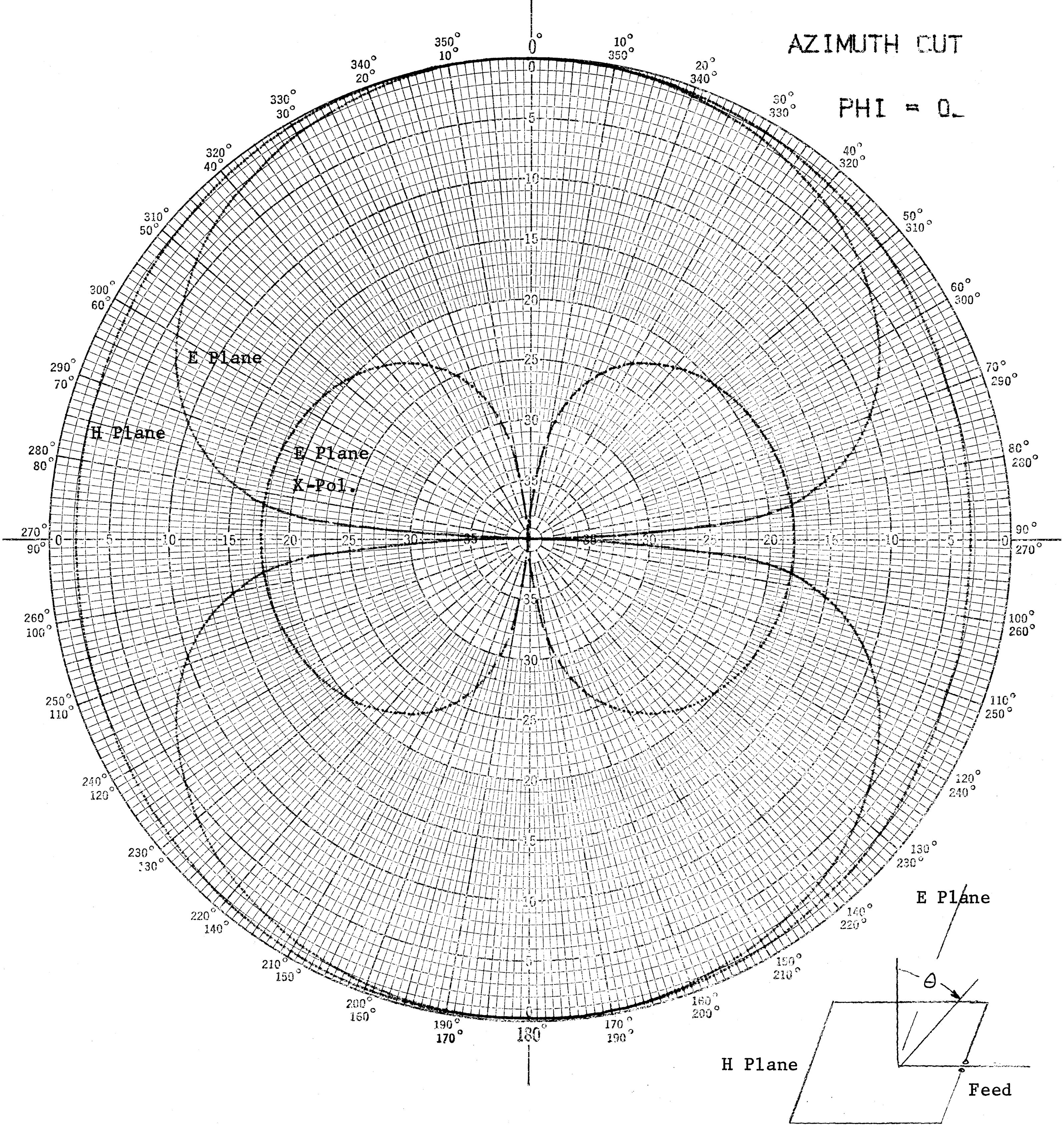
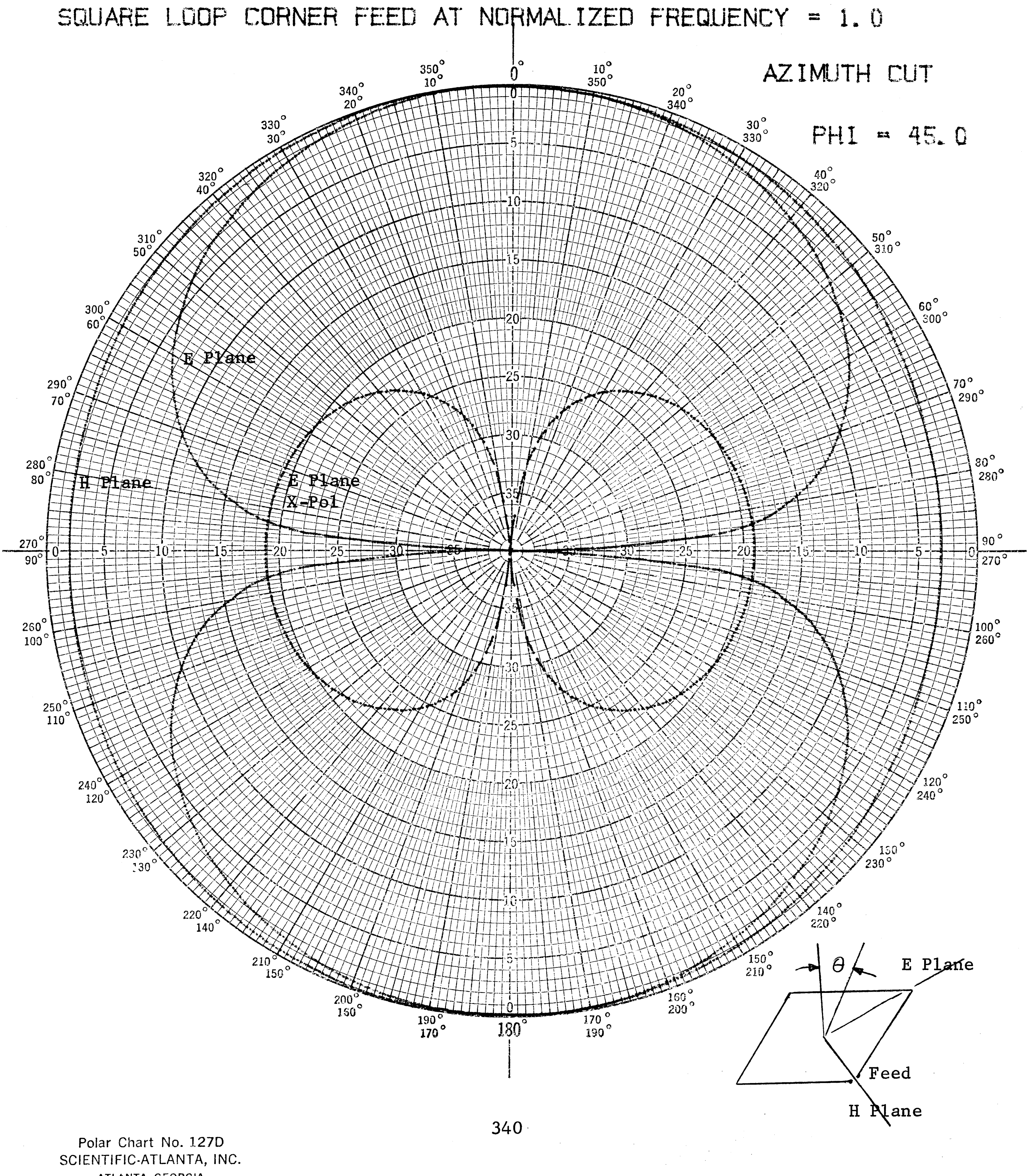

Input Impedance of One Wavelength Loop
Normalized Frequency
0.9
1.0
1.1
1.2
Resistance Ohms
240
220
200
180
160
140
120
100
80
60
40
20
Reactance Ohms
240
200
160
120
80
40
0
-40
-80
-120
-160
-200
-240
Resistance
Reactance
1/26/81 TAM
Normalized Frequency
0.9
1.0
1.1

SQUARE LOOP AT NORMALIZED FREQUENCY = 1.0




SQUARE LOOP CORNER FEED AT NORMALIZED FREQUENCY = 1.0
AZIMUTH CUT
PHI = 45.0
E Plane
H Plane
E Plane
X-Pol
θ
E Plane
Feed
H Plane
Polar Chart No. 127D
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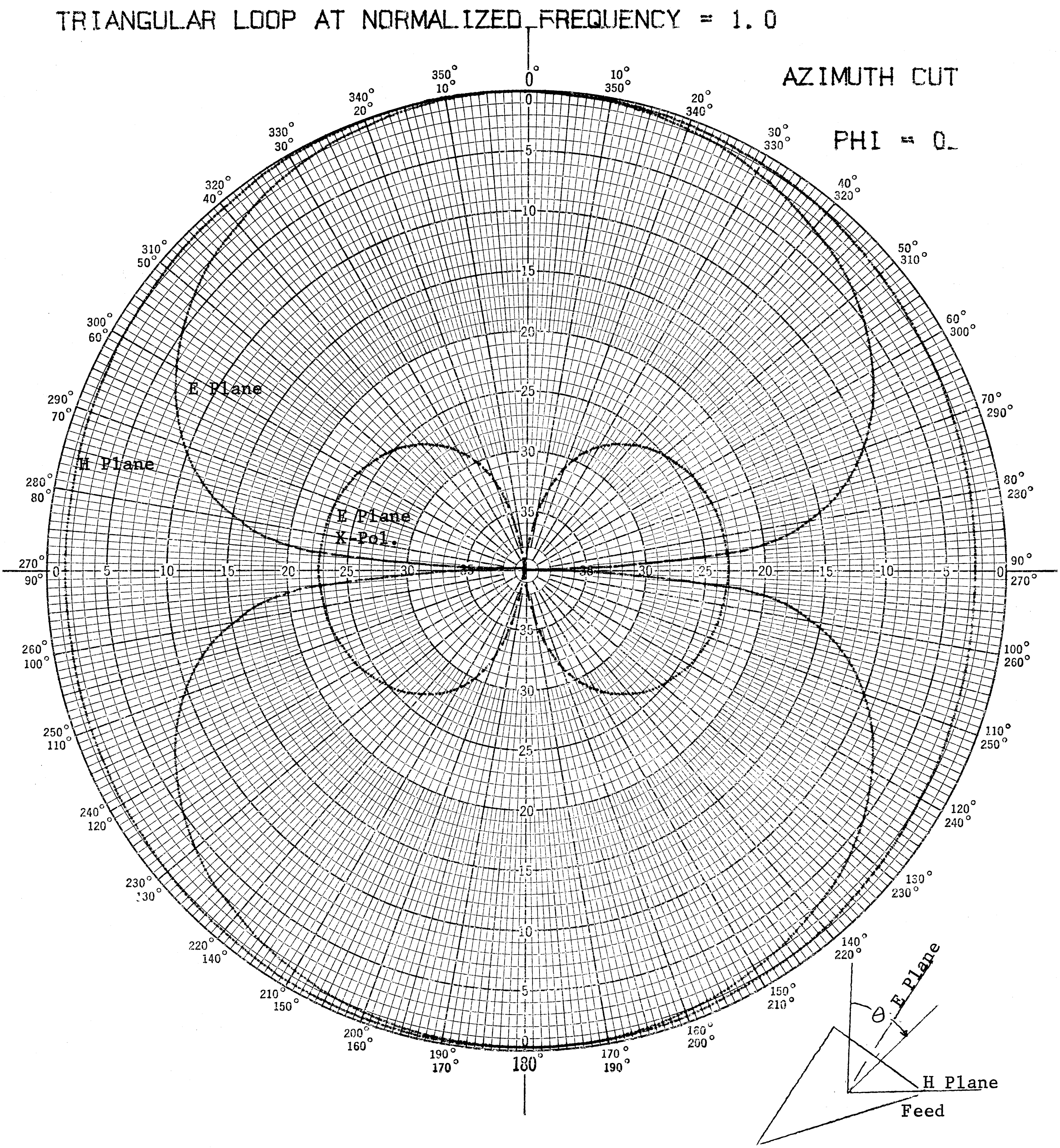

TRIANGULAR LOOP AT NORMALIZED FREQUENCY = 1.0
AZIMUTH CUT
PHI = 0
E Plane
H Plane
E Plane
X-Pol.
θ
E Plane
H Plane
Feed

QUAD ANTENNA

These squares, loops, or triangles can be arrayed with parasitic loops to get a undirectional pattern. The usual configuration uses one of the loops as a reflector. This antenna is called a quad antenna by the amateur radio people who invented it and are the chief users. The antenna is traditionally a square loop; hence the name quad. There are no tables of the mutual impedance of loops, so the antenna, like other Yagi type antennas, is designed experimentally. Using a moment method solution, the following design was found.

Feed:	1.05 Wavelength perimeter
Reflector:	1.07 Wavelength perimeter
Spacing:	0.1 Wavelength

The pattern at center frequency is plotted on page 343. The antenna has 8 dB directivity although the F/B is only 8.9 dB. When the frequency is increased the F/B improves but the gain drops. The pattern at normalized frequency 1.04 is plotted on page 344. At this frequency the F/B is 20.6 dB but the gain has dropped to 7.2 dB. The following table has been prepared from analyzing this design.

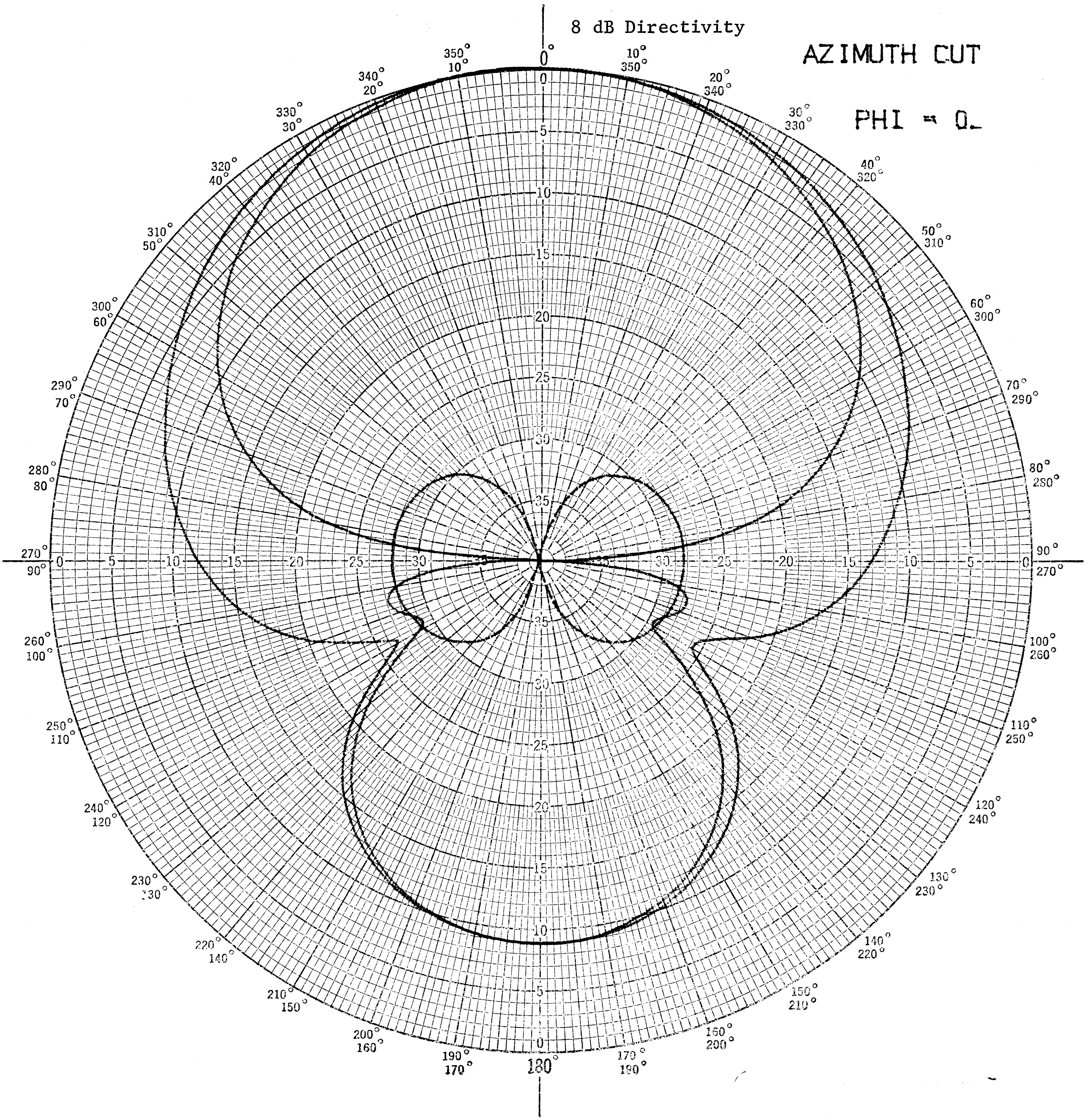
Normalized Frequency	Gain	F/B	Input Impedance	
.95	3.9	-3.0	19.9	-87.4
.98	7.4	3.0	23	-8.4
1.00	8.0	8.9	39.3	49.9
1.02	7.6	16.7	71	104.2
1.04	7.2	20.6	113.6	145.8
1.06	6.9	15.2	157.4	172.7
1.08	6.6	12.0	196	190
1.10	6.4	10	228	204

The gain peaks at the center frequency while the F/B peaks at a higher frequency. Although the gain is dropping, the antenna has good gain and reasonable F/B over a 10% frequency band. The input impedance varies quite rapidly and it would be difficult to achieve a large impedance bandwidth. The gain to a feeding transmission line would fall off faster than shown because of the mismatch losses.

The antenna is inherently narrow band but it is well suited to the narrow bandwidths used in amateur radio. The antenna is usually constructed from wire stretched between a cross of dielectric poles with the various higher frequency bands inside the outer loop. High frequency antennas have been constructed using metal loops. These loops can be riveted to a metal rod at the point of the virtual short circuit which is located opposite the feed point. Any point in the antenna or circuit which is a virtual short circuit can be shorted without effecting the proper performance. In some cases the short will eliminate unwanted modes.

Quad Antenna Feed 1.05 Wavelengths, Reflector 1.07 Wavelengths

Spacing 0.1 Wavelengths   at Center Frequency



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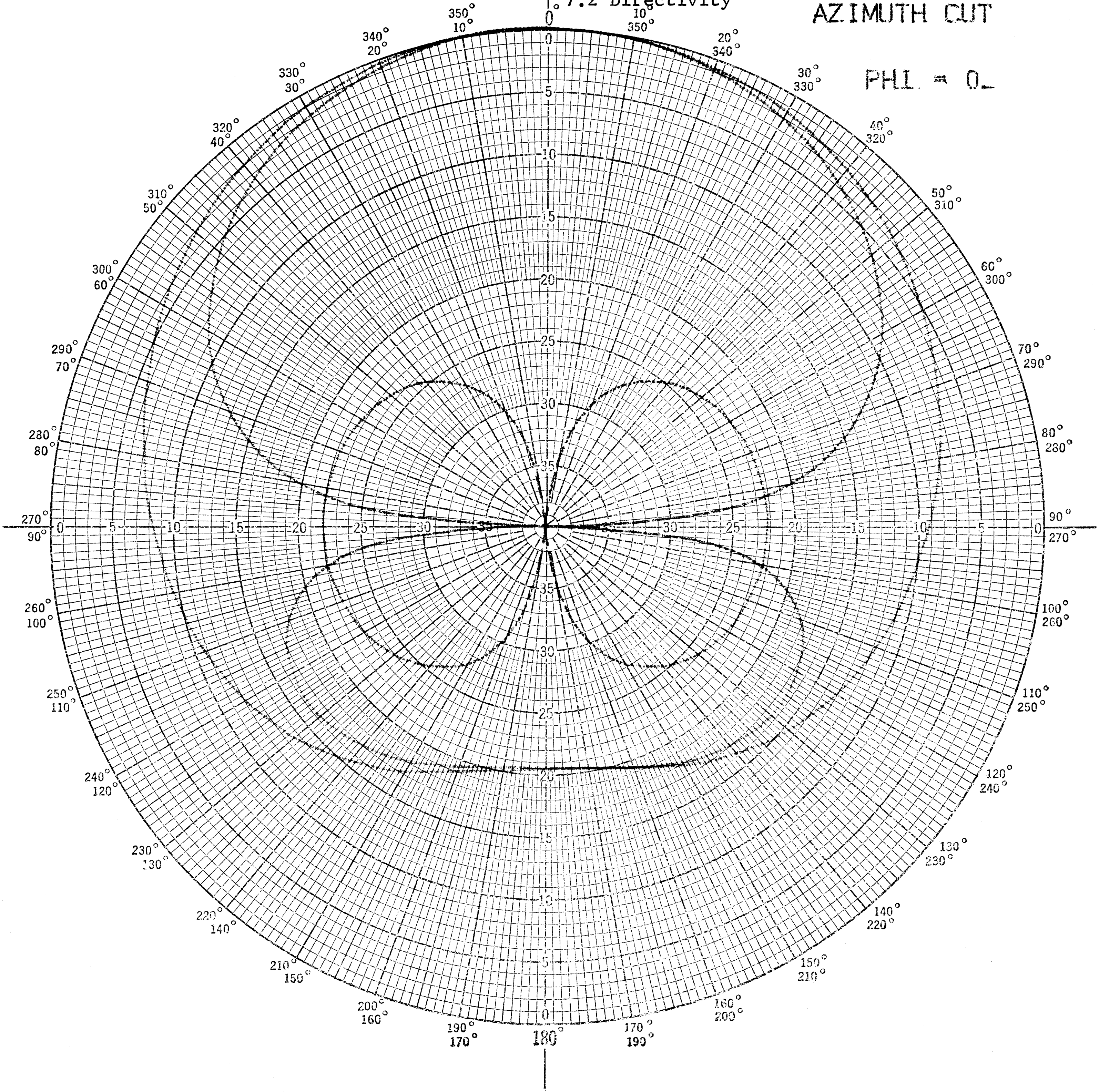
Quad Antenna Feed 1.05 Wavelengths, Reflector 1.07 Wavelengths

Spacing 0.1 Wavelengths 1.04 Normalized Frequency

7.2 Directivity



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LOOP PARASITIC ELEMENT

The loop can be used with dipole elements as a parasitic element. A parasitic loop can be used to eliminate high back lobes of an antenna which normally has good F/B such as a log periodic dipole. The loop can be used to increase the low frequency response of the log periodic dipole antenna by being a reflector to the lower dipoles of the antenna. When placed under a dual linearly polarized antenna, the loop will be a reflector for both polarizations.

A resonant length dipole (.47 wavelengths) was placed over a loop at a quarter-wave distance and the circumference of the loop was varied. The following table was generated.

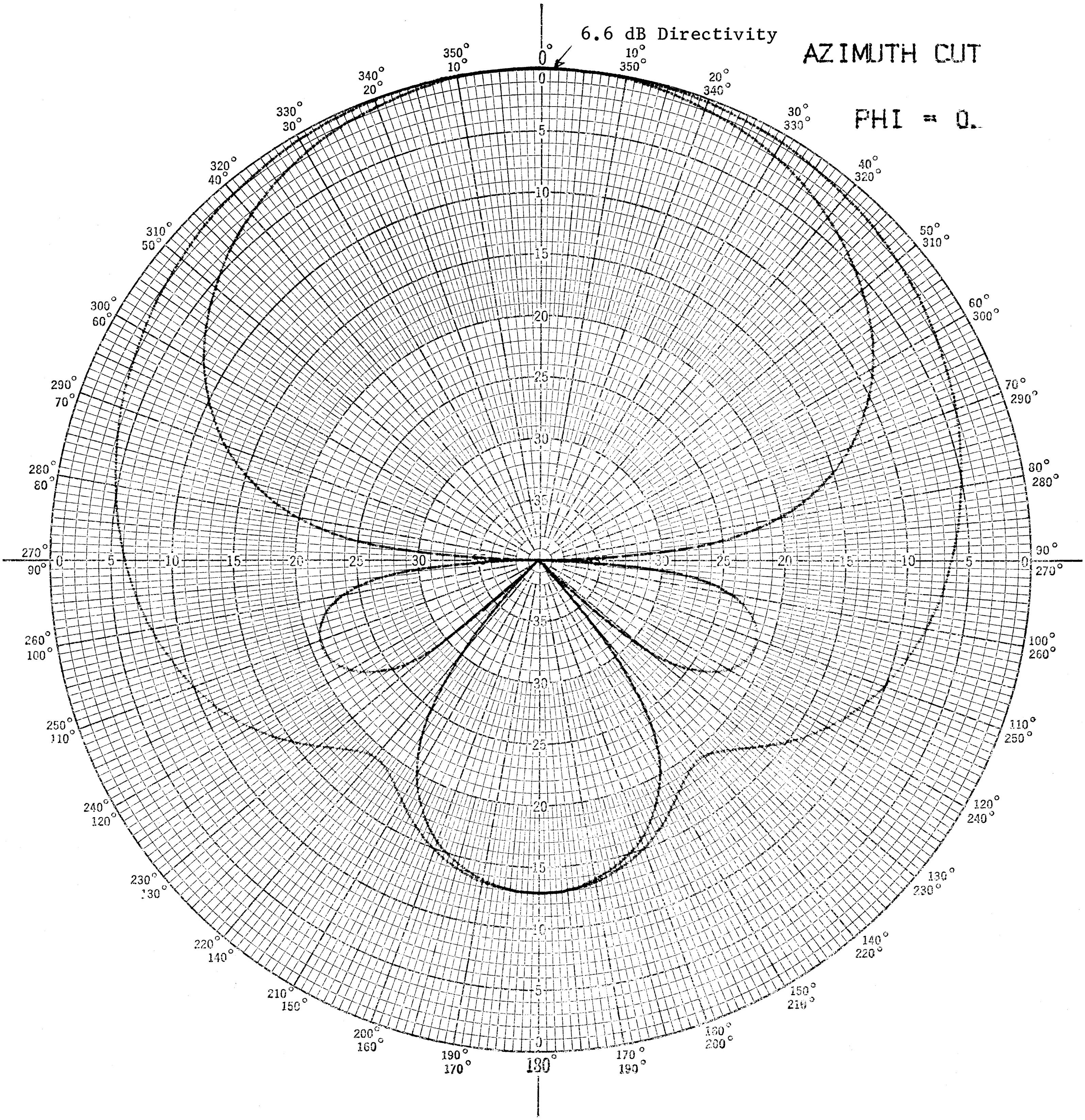
Loop Circumference	Gain	F/B	Input Impedance Real	Input Impedance Imaginary
1.0	5.7	1.4	46.7	20.8
1.05	6.9	5.8	60.1	34.7
1.10	6.6	13.0	80.7	32.1
1.15	5.9	21.7	89.6	19.6
1.20	5.3	15.5	90.1	10.0
1.25	4.8	11.6	88.4	4.2

The patterns for a loop circumference of 1.10 and 1.15 wavelengths are plotted on pages 346 and 347. Both of these would be suitable designs. This is easier to construct than the quad reflector antenna and has no cross polarization response. Linear elements can be used as directors with the quad antenna to simplify the construction instead of all loops. The frequency response of the antenna with a 1.15 wavelength loop in given in the following table.

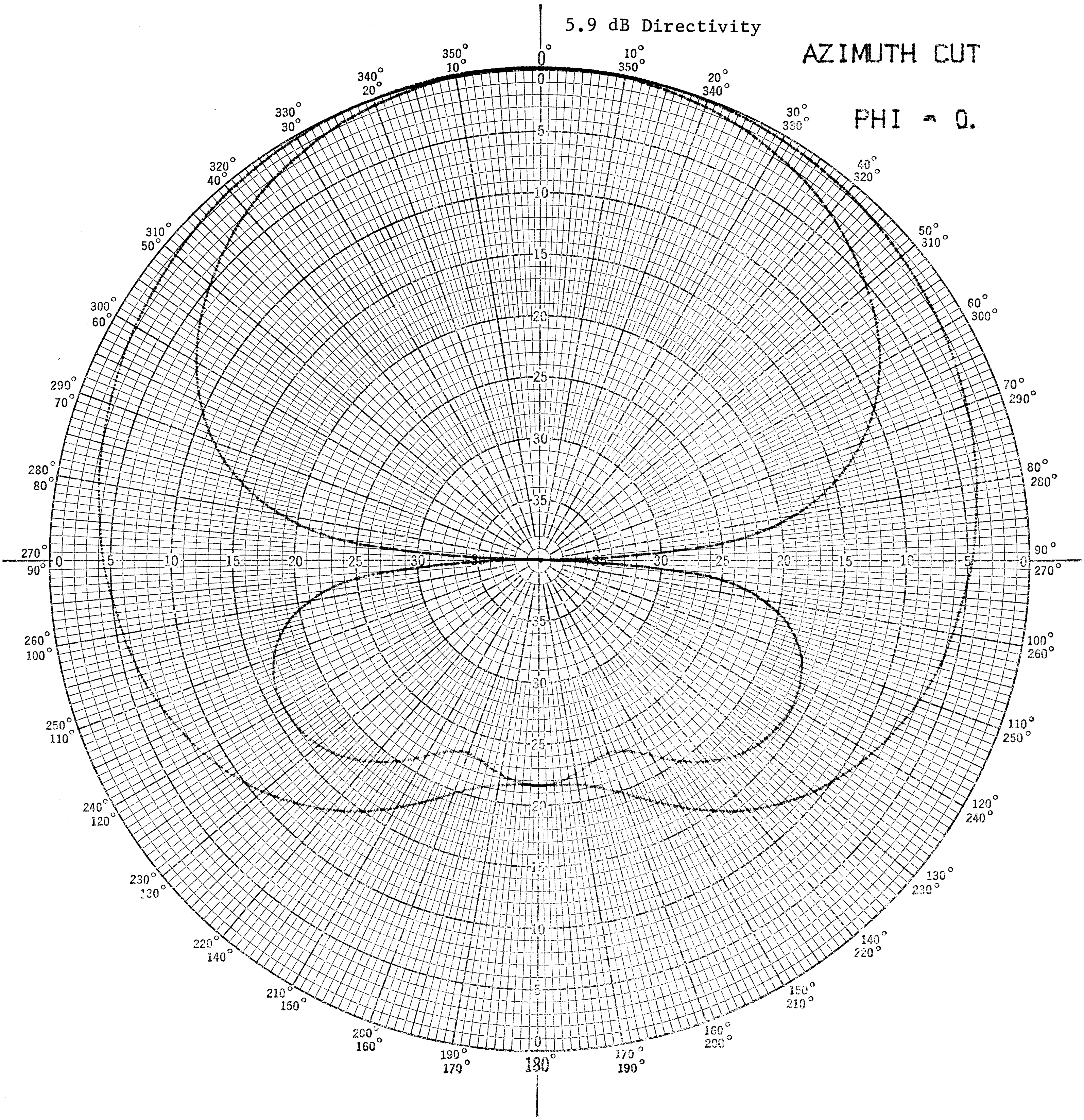
Normalized Frequency	Gain	F/B	Input Impedance Real	Input Impedance Imaginary
0.90	6.8	4.2	35.7	-51.1
0.95	6.8	12.2	66.0	-6.1
1.00	5.9	21.7	89.6	19.6
1.05	5.1	14.4	102.6	45.4
1.10	4.5	10.4	113.1	75.1

The antenna maintains a good front to back ratio over a 15% bandwidth. Although the gain of this antenna is not quite that of the quad antenna on page 342, the input impedance does not vary as rapidly. It would be easier to obtain a reasonable input match over a larger bandwidth with this antenna.

0.472 Wavelength Dipole Quarter-wave over 1.10 Wavelength Circumference Loop



0.472 Wavelength Dipole Quarterwave over 1.15 Wavelength Circumference Loop



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