Conversion of Single Arm Spiral Antenna Measurements

Abstract
Multiple arm spiral antennas, generally four or more arms, can be fed in multiple modes that produce different antenna patterns. The patterns and S-parameters of the antenna operating in different modes can be found by combining a weighted sum of measurements made on individual arms. The frequency range of good efficiency can be determined from S-parameter measurements before the antenna patterns and gain are measured. Individual arm measurements isolate construction problems that would be masked by a feed network.

Introduction [1,2]
Before a spiral antenna is connected to a feed network most of its characteristics can be

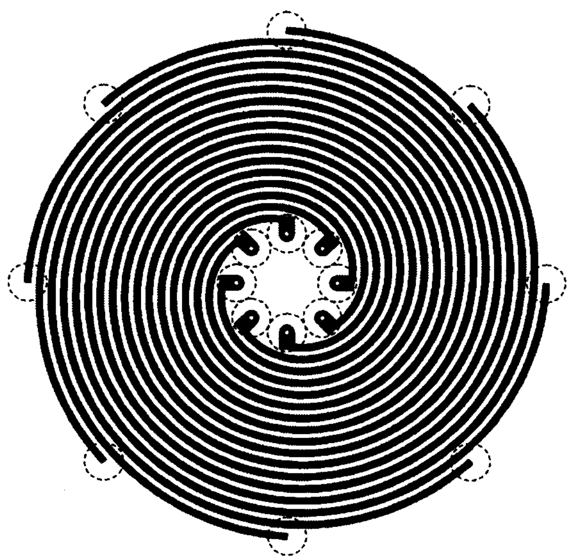

Figure 1. Eight-Arm Spiral with Each Arm Fed from a Coaxial Cable on the Inside and Outside

determined from measurements made on single arms. These measurements greatly increase our knowledge of the operation of the antenna and can be used to improve it because problems are isolated. Figure 1 illustrates the face of an eight-arm spiral antenna. Each arm of the antenna has been connected to coax transmission lines both on the inside and outside ends. The wrap of the spiral in Fig. 1 radiates right hand circular, RHC, polarization when fed from the center and LHC polarization when fed from the outside. The antenna can radiate LHC polarization for the inner feeds as the antenna ports when the feeding signals are improperly phased or when signals reflect from the outer ends of the spiral. A feed network splits the power into the arms with proper phases and equal amplitudes to produce the various modes of radiation. This feed network is sometimes referred to as a beamformer and provides a separate port for each mode.

This article shows how to combine measurements made on individual arms of the spiral to find the patterns and impedance properties when it is operating in various modes. The analysis considers the spiral antenna as an array and uses the active pattern of a single arm. We sum the patterns of the individual arms weighted by the mode coefficients to find the pattern in a given mode. In a similar manner the active impedance of the spiral is found from a combination of the input reflection and the weighted sum of the mutual coupling between the arms.

We can use either measurements made on the individual arms or measure a single arm and assume the antenna is symmetrical and use array theory to find the response of the multiple arm antenna when fed with a beamformer feed network. This technique eliminates the need to build the feed network during initial stages of development and

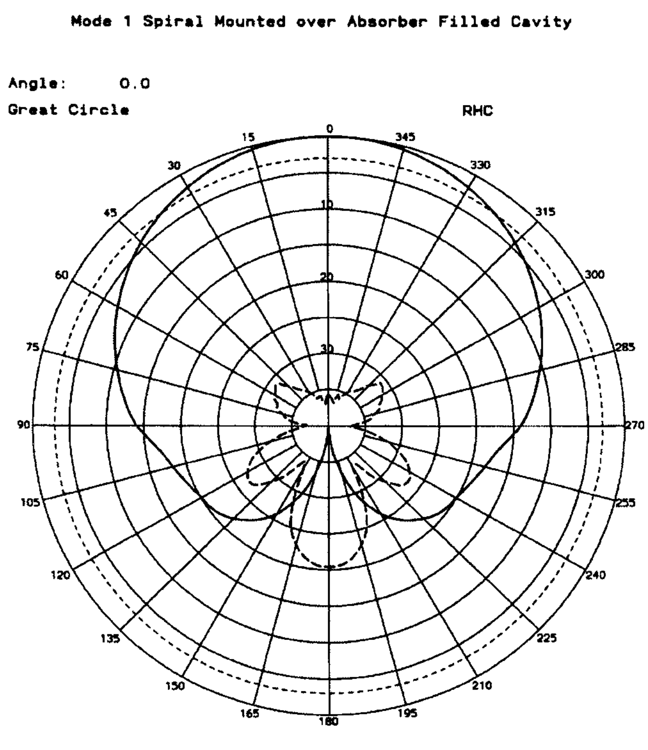

Figure 2. Pattern of Spiral Antenna, Mode 1

isolates construction problems. Furthermore, we can evaluate every possible spiral antenna mode from the single arm measurements.

Spiral Modes

The output signals of an N-port beamformer can be decomposed into N-modes that correspond to N-modes of spiral antenna radiation. The spiral modes have equal amplitudes and a linear progression of phase on the arms. The progression of phase is given by Mode ϕ where ϕ is the CCW rotation around the spiral arms when looking at the face. Mode is an integer with Mode = 1 corresponding to RHC polarization. Figures 2, 3, and 4 show the radiation patterns of the three RHC modes for an eight-arm spiral over an absorber-loaded cavity when fed by a perfect beamformer. Although the outputs of the beamformer can be decomposed into only N-modes, the spiral antenna can radiate any number of modes for a given set of feeding coefficients. The finite value of cross-polarization, the dashed curve in the figures, radiated in the forward hemisphere of the pattern is caused by extra modes radiated by the antenna. If we rotate the spiral CCW about its axis by ϕ, the radiation phase will change linearly by $-$Mode ϕ when radiating in only a single mode. Imperfections in the construction of the spiral or the impedance mismatches cause the undesired modes of radiation. These modes are a Fourier series expansion of the radiation from the spiral antenna which uses the spiral modes as the basis functions.

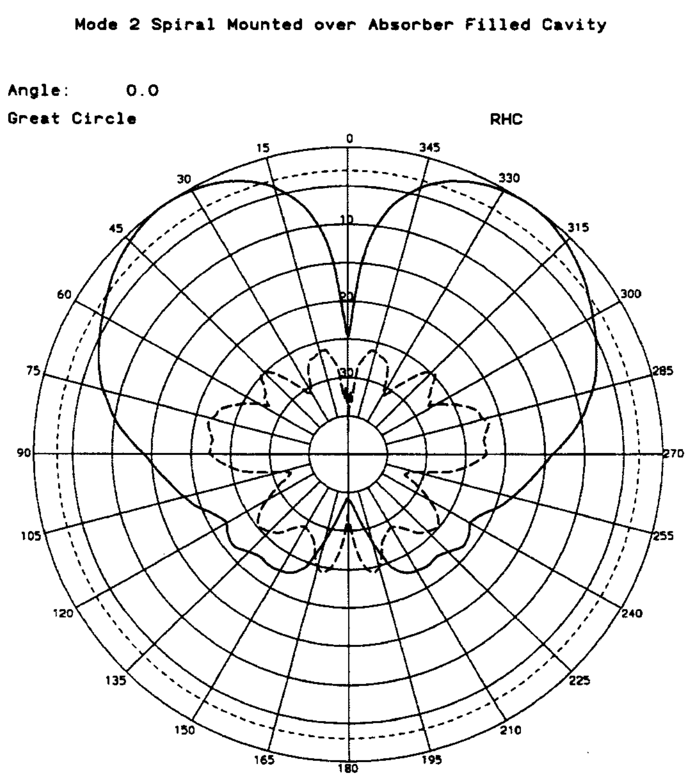

Figure 3. Pattern of Spiral Antenna, Mode 2

Spiral Mode Feeding Coefficients

An N_{arm} arm spiral can be fed with equal power into the arms in N_{arm} possible modes while the actual spiral when fed from a beamformer can radiate any number of modes. An arbitrary feeding set of amplitudes and phases on the arms can be decomposed into a sum of these orthogonal modes. The feeding coefficient on arm N for the various modes is given by

$$V_N = \frac{\exp(-j2\pi Mode(N-1)/N_{arm})}{\sqrt{N_{arm}}} \qquad (1)$$

where the arms are numbered CCW when looking at the spiral face, Fig. 1. The normalized power into each arm is $V_N V^*_N$ because V_N is divided by the square root of the number of arms. Mode = 1 corresponds to RHC which has its pattern peak broadside to the spiral face as shown in Fig. 2. Table 1 gives the excitation phases for the various

Mode 3 Spiral Mounted over Absorber Filled Cavity

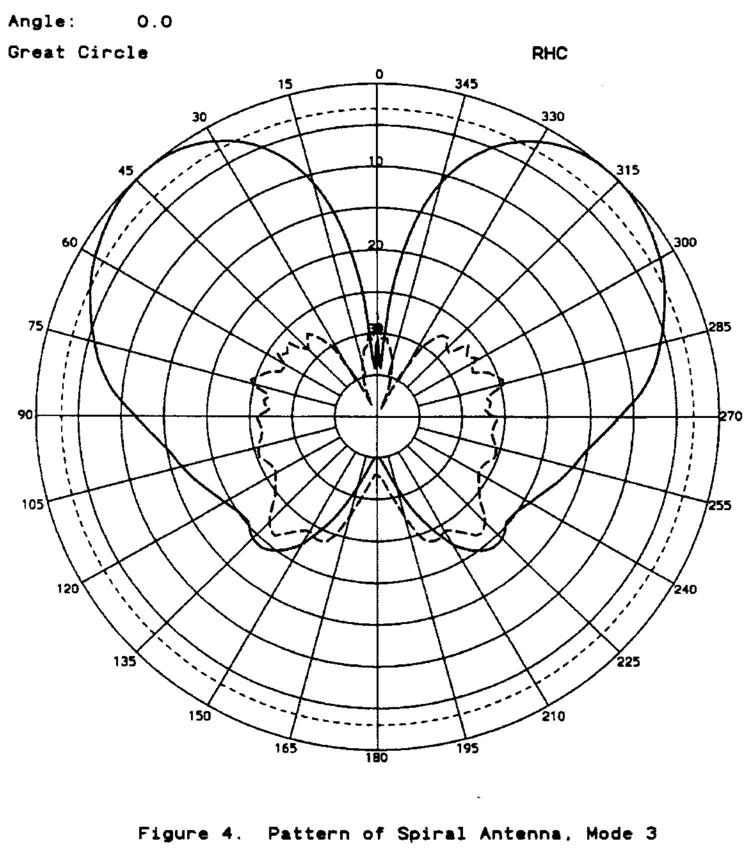

Figure 4. Pattern of Spiral Antenna, Mode 3

modes of a four-arm antenna. Mode 2 can be either RHC or LHC polarization and will depend on the wrap direction of a spiral.

Table 1 Four Arm Spiral Antenna Excitation Phases

Mode	Polarization	Arm 1	Arm 2	Arm 3	Arm 4
0 or 4	RHC/LHC	0	0	0	0
1	RHC	0	-90	180	90
2	RHC/LHC	0	180	0	180
3 or –1	LHC	0	90	180	-90

Multiple arm antennas can radiate different patterns by changing the excitations of the arms. Spirals radiate circularly polarized waves whose far field pattern phase changes linearly when the antenna is rotated about the spiral axis. The phase rotation is given by -Mode ϕ for a rotation ϕ (CCW) about the spiral axis. The feed phases for an eight-arm spiral antenna are given in Table 2.

Table 2 Eight-Arm Spiral Antenna Excitation Phases

Mode	Polarization	Arm 1	Arm 2	Arm 3	Arm 4	Arm 5	Arm 6	Arm 7	Arm 8
0 or 8	RHC/LHC	0	0	0	0	0	0	0	0
1	RHC	0	-45	-90	-135	180	135	90	45
2	RHC	0	-90	180	90	0	-90	180	90
3	RHC	0	-135	90	-45	180	45	-90	135
4	RHC/LHC	0	180	0	180	0	180	0	180
5 or –3	LHC	0	135	-90	45	180	-45	90	-135
6 or –2	LHC	0	90	180	-90	0	90	180	-90
7 or –1	LHC	0	45	90	135	180	-135	-90	-45

Array Analysis of Spiral

Consider a normal array where the elements are spaced on some grid. When each antenna radiates, part of its radiation is received by nearby elements. The antennas that receive this power absorb part of it and re-radiate the rest. The power received by the antennas changes the effective distribution in the array. When the antenna elements can only be excited in a few modes, as is the case for small multi-arm antennas, the net effect is to change the amplitude and phase distribution on the array. Two approaches can be used to predict this effect.

A mutual coupling matrix relates the excitations at the feed ports to the effective pattern radiated by each element. Mutual coupling changes the distribution. We can measure the mutual coupling using a network analyzer and express it as a scattering parameter (S-parameter), impedance or admittance matrix. By inverting the matrix we can find the effective radiation currents from the excitation voltages (or voltages from currents, etc.).

This gives us the new excitations of the elements and we can predict the radiation patterns. For an arbitrary array element the more general technique of the active pattern is an easier approach.

The active pattern method uses measurements of the antenna pattern of each element when it is imbedded in the array. The nearby antennas are loaded with the characteristic impedance of the feed network. The effective radiation from the driven element is the sum of the radiation from the element and the radiation from currents excited on the nearby elements. We measure the effective pattern of the element in the array and assume, in many cases, that all the elements have the same pattern even though elements near the edge will have a different pattern. This allows the calculation to be separated into the product of the array factor and the element factor. A spiral antenna has a rotational symmetry to its arms which requires the pattern of a single arm to be rotated to the position of the other arms before we can add its radiation. We cannot separate it into the product of an array factor and an element factor. Each element of the array (spiral arm) has the same pattern but each is pointing in a different direction.

Spiral Pattern Measurements

The characteristics of multiple arm spirals can be determined by measurements made on the individual arms. By connecting a coax line to each arm as shown in Fig. 1 they can be measured separately when the other ports are loaded. The patterns can be determined by either combining the patterns of the individual arms or the pattern of a single arm can be duplicated and rotated to the position of the other arms and combined using the amplitude and phases of a beamformer in the various possible modes.

We consider the multiple-arm antenna as an array. For a spiral we connect the center conductor of a coax cable to the ends of each arm of the spiral. We connect the coax outer conductors together for the inner feeds of the spiral. The outside feed outer conductors are connected to a grounding ring. The mutual coupling between the arms is high because they are closely wrapped. We add the responses from each arm in the same manner as an array. We measure the pattern of each arm separately with the others loaded or use the measurement of the active pattern of a single arm and use an array calculation to predict the pattern of the antenna when excited in various modes. Instead of spacing the elements in the X-Y plane to form an array, we rotate the pattern of the single-arm measurement.

If we measure the pattern of each arm separately, it does not matter what type of positioning is used during the measurements. We do not need to rotate the patterns, but only add them after multiplying each one by the mode coefficient for that arm. If we only measure the pattern of a single arm, assume all the arms are identical, and rotate the pattern to the position of each arm, it is convenient to measure the scans about the axis of the spiral. In this case a cyclic rotation can be used on the scans to find the pattern of each arm. If routines are available to rotate the pattern and properly account for rotation of polarization, then it does not matter what type of antenna positioning is used.

Mode Amplitudes from Measurements

Although the spiral arms can only be fed in N_{arm} orthogonal modes, the spiral can radiate any order mode. Errors in the beamformer feed network outputs will produce a combination of modes that can be decomposed into N_{arm} modes for each input port. Given the output waves $\mathbf{b_n}$ of the beamformer for a given input port, the division between the modes is given by

$$\mathbf{b_m} = \sum_{\mathbf{n}} \mathbf{V^*_{n,m} b_n} \qquad (2)$$

where $\mathbf{V^*_{n,m}}$ are the complex conjugate of the mode excitations given by equation (1) on the n-th arm for mode m. The relative power in each mode radiated by the antenna can be found from the following integral,

$$\int \left(\left| \int_0^{2\pi} E_R \exp(jMode\phi)\, d\phi \right|^2 + \left| \int_0^{2\pi} E_L \exp(jMode\phi)\, d\phi \right|^2 \right) \sin\theta\, d\theta \qquad (3)$$

where E_R and E_L are the RHC and LHC voltage components. The pattern can contain higher order modes than the limit determined by the number of arms. The conic patterns are decomposed into a Fourier Series using the spiral modes as the expansion functions with coefficients given by the inner integrals of equation (3). We can normalize by summing the radiated power over a large range of modes since the radiated power drops as the mode number increases to determine the efficiency of the lower order modes. Note that the inner integral needs to be taken over angles about the axis of the spiral, that is, the upper axis on a model tower positioner or ϕ axis.

S-Parameter or Impedance Measurements

The input impedance for each mode can be determined by combining the reflection coefficient and the mutual coupling to the other arms weighted by the mode coefficients. Because we analyze the multiple-arm antenna as an array, we can use the active impedance for the input impedance of each mode. If the array impedance is expressed using mutual impedance, then the array can be represented with the following matrix equation

$$\mathbf{V} = \mathbf{Z\, I}$$

where $\mathbf{V}$ is the vector of the voltages, $\mathbf{Z}$ is the matrix of self and mutual impedance, and $\mathbf{I}$ is the vector of currents. The active impedance of the first element is calculated by

$$Z_1 = Z_{11} + Z_{12}\frac{I_2}{I_1} + Z_{13}\frac{I_3}{I_1} + \ldots + Z_{1N}\frac{I_N}{I_1}$$

We can formulate the problem by using S-Parameters: $\mathbf{b} = \mathbf{S\, a}$ where the vector $\mathbf{a}$ is the combination of the input waves and the vector $\mathbf{b}$ is the combination of the reflected waves from the spiral ports. We measure these parameters directly on a network analyzer with $\mathbf{S}$ containing the mutual coupling terms. In a spiral we can assume symmetry and need only measure from one input port to all other ports. If we feed port 1 and load all other ports in the characteristic impedance of the S-Parameter system, the input reflection coefficient becomes

$$\Gamma_1 = \frac{b_1}{a_1}$$

We can formulate the active reflection coefficient in a similar manner as the active impedance.

$$\Gamma_1 = \frac{b_1}{a_1} = S_{11} + S_{12}\frac{a_2}{a_1} + S_{13}\frac{a_3}{a_1} + \ldots + S_{1N}\frac{a_N}{a_1}$$

The matrix terms S_{ij} are the measured mutual coupling terms between the arms of the antenna. The values $\mathbf{a}_i$ are all the same magnitude with the phases given by the mode voltages. We convert the reflection coefficient to input impedance by using the familiar relationship

$$Z_{in} = Z_0 \frac{1+\Gamma_1}{1-\Gamma_1}$$

where Z_0 is the coaxial characteristic impedance (normally 50 Ω). The input impedance depends on the mode phases. By symmetry we can see that the input impedance at all ports is the same for the modes given above. If the impedance we measure at the ports are not similar, then a fabrication problem has occurred.

If we place connectors on the output of each arm of the spiral, we can measure the power lost in the loads of the antenna. These loads prevent reflections from the spiral end which would radiate the opposite sense of circular polarization when the wave travels back to the input. We denote the output ports as $N_{arm} + 1$ through $2N_{arm}$ in a CCW order. In a spiral we can assume symmetry and only measure from one input to all the loaded ends. The power lost in the first load can be found from

$$b_{N+1} = S_{N+1,1}a_1 + S_{N+1,2}a_2 + S_{N+1,3}a_3 + \ldots + S_{N+1,N}a_N$$

Note that there is rotation of output ports. The coupling from input 2 to the end of the first arm $S_{N+1,2}$ is given by the measured term $S_{2N,1}$. The general term $S_{N+1,j}$ is found from the measured coupling $S_{2N+2-j,1}$. The output voltage wave into the load on arm 1 depends on the excitation mode. Because the power will be so low when the wave arrives at the loads on the ends of the arms, the load mismatch has little effect on the input impedance. In a properly sized spiral most of the power will radiate and very little will be lost in the loads on the ends of the arms.

Efficiency from S-parameters

We can use measurements using a network analyzer to determine the efficiency of a spiral in its various modes over the frequency range of the antenna. The antenna distributes the input power into: 1) radiation, 2) reflected power loss, 3) power into the loads, 4) losses in the transmission line formed by the spiral to carry power to the active region, and 5) radiation into the cavity absorber, if any. We can measure the radiated power by using an antenna range, but before that step is taken the other losses can be measured or estimated.

In the design of the antenna the losses in the transmission line should be estimated. The spiral can be considered as a coplanar transmission line and the losses estimated through what could be a very long transmission line to the active region. For this calculation it is assumed that the active region is centered where the circumference of the antenna is Mode*λ. Etching a spiral with a low growth rate or the use of substrates with poor loss tangents can lead to large losses. The equations above give the reflected power loss ($1 - |\Gamma|^2$) and the power into the loads when fed by each mode. Plots of the spiral losses versus frequency for various excitation modes quickly reveal the frequency range of good performance.

Conclusions

Both the pattern and impedance properties of a multiple-arm spiral antenna can be found from measurements on single arms of the spiral. By testing the antenna without the feed network that generates the various modes, construction problems are easily isolated. The analysis of the spiral as an array gives an effective means of determining the spiral antenna characteristics independent of the beamformer and aids the impedance matching of the antenna to the beamformer feed network. S-parameter measurement of both the mismatch and the power delivered to the loads on the arms provides a means of estimating efficiency and the operating frequency range.

[1] Corzine, Robert G. and Joseph A. Mosko, **Four-Arm Spiral Antennas**, Artech, Norwood, MA, 1990.

[2] DuHamel, R. H. and J. P. Scherer, **Antenna Engineering Handbook, 3rd ed.,** R. C. Johnson, ed., Chapter 14, McGraw-Hill, New York, 1993.